

Chapter One: **Pre-Geometry**

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Recall:

Solving Basic Equations: The goal of solving any equation is to get the variable by itself. This is known as *isolating the variable*.

Golden Rule of Algebra: "Do unto one side of the equal sign as you will do to the other..."

****Whatever you do to one side of the equal sign, you MUST do the same thing on the other side****

Steps to Solve:

- 1) Identify the given operation
- 2) Figure out the opposite operation
- 3) Use the opposite operation to isolate the variable
- 4) Box your final answer

Here are some examples to "refresh" your memory:

$$\begin{array}{r}
 1) \quad 2x + 11 = 19 \\
 \underline{-11 \quad -11} \\
 2x = 8 \\
 \underline{\quad 2 \quad 2} \\
 \boxed{x = 4}
 \end{array}$$

$$\begin{array}{r}
 2) \quad 3(x - 5) = 12 \\
 3x - 15 = 12 \\
 \underline{+15 \quad +15} \\
 3x = 27 \\
 \underline{\quad 3 \quad 3} \\
 \boxed{x = 9}
 \end{array}$$

$$\begin{array}{r}
 3) \quad 4x + 2 + 3x = 9 \\
 7x + 2 = 9 \\
 \underline{-2 \quad -2} \\
 7x = 7 \\
 \underline{\quad 7 \quad 7} \\
 \boxed{x = 1}
 \end{array}$$

$$\begin{array}{r}
 4) \quad 5x - 2 = 12x + 19 \\
 \underline{-12x \quad -12x} \\
 -7x - 2 = 19 \\
 \underline{+2 \quad +2} \\
 -7x = 21 \\
 \underline{-7 \quad -7} \\
 \boxed{x = -3}
 \end{array}$$

Now let's try some examples. Make sure to show all work.

1) $9x - 11 = -38$	2) $3x + 15 + 4x = -13$	3) $5 = 4(x + 3) - 5x$
4) $-9x + 34 = 79$	5) $4x - 7 = -23$	6) $101 = 69 - 7x + 3x$

$$7) 15 = \frac{1}{3}x - 9$$

$$8) 11x - 3 = 7x + 17$$

$$9) 4x + 4 = 2(x - 3)$$

$$10) 8x + 6 = 3(5x + 16)$$

$$11) 5(6 - x) = -3(x + 2)$$

$$12) 3(7x + 5) + 2(4x - 9) = 55$$

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Solving Equations 1A HW

1) $x = -3$

2) $x = -4$

3) $x = 7$

4) $x = -5$

5) $x = -4$

6) $x = -8$

7) $x = 72$

8) $x = 5$

9) $x = -5$

10) $x = -6$

11) $x = 18$

12) $x = 2$

Factoring

1. Greatest Common Factor

The first type of factoring is known as the **Greatest Common Factor**. The reason why it is called this is because we have to look for the **highest number** and **variable** that the terms have in **common**. When choosing the GCF for the variables, if all the terms have a common variable, take the one with the **lowest** exponent.

- The greatest common factor is the largest factor between two numbers.

$$\begin{array}{l} \underline{12} = 1, 2, 3, 4, \textcircled{6}, 12 \\ \underline{18} = 1, 2, 3, \textcircled{6}, 9, 18 \end{array}$$



Consider: $10x^2y^3$ and $15xy^2$

The greatest common factor is $5xy^2$.

The largest factor of 10 and 15 is 5.

The highest power of x that is contained in both terms is x .

The highest power of y that is contained in both terms is y^2 .

Examples:

1) $9x^4 + 3x^3$

2) $12x^5 + 16x^4 + 24x^3$

3) $18x^7 - 27x^5 + 36x^3$

4) $5c^3 - 25c^2 + 10c$

5) $28x^3y^2z + 63x^5y^4z - 56x^8y^3$

6) $8x(x+5) - 11(x+5)$

****REMEMBER****

In any type of factoring, we **ALWAYS**
have to look for a GCF first!

7) $9x(x+1) + 7(x+1)$

8) $4x(x-6) - 9(x-6)$

9) $3x^2 + 3x - 4x - 4$

10) $5x^2 + 20x - 3x - 12$

2. DOTS

The **Difference of Two Squares** or DOTS, is in the form $a^2 - b^2$. DOTS was created when two conjugate pairs (opposite signs) binomials are multiplied together. Therefore, all DOTS problems can be factored to be in the form $(a + b)(a - b)$.

Factor: $x^2 - 9$

Both x^2 and 9 are perfect squares. Since subtraction is occurring between these squares, this expression is the **difference of two squares**.

What times itself will give x^2 ? The answer is x .

What times itself will give 9? The answer is 3.

These answers could also be negative values, but positive values will make our work easier.

The factors are $(x + 3)$ and $(x - 3)$.

Answer: $(x + 3)(x - 3)$ or $(x - 3)(x + 3)$ (order is not important)



Be careful!!

This process of factoring does

NOT apply to

$a^2 + b^2$.

Examples:

1) $x^2 - 25$

2) $m^2 - 16$

3) $x^2 - 256$

4) $121 - a^2$

5) $b^2 + 49$

6) $3x^2 - 48$

7) $g^4 - 81$

8) $a^2 b^2 - 144$

Directions: Answer the following questions completely. Please make sure to show all work that was shown in class!

1) $14x^9 - 7x^7 + 21x^5$

2) $32x^6 - 12x^5 - 16x^4$

3) $16x^5y^2 - 8x^4y^3 + 24x^2y^4 - 32xy^5$

4) $24b^{11} + 4b^{10} - 6b^9 + 2b^8$

5) $11x^3y^3 + 121x^2y^2 - 88xy$

6) $75a^5 + 15a^4 - 25a$

7) $132a^5b^4c^3 - 48a^4b^4c^4 + 72a^3b^4c^5$

8) $16x^5 + 12xy - 9y^5$

9) $3x(x + 1) - 4(x + 1)$

10) $x^2(x + 5) + 2(x + 5)$

11) $7x^2 - 14x + x - 2$

← HINT: Split these down
the middle! →

12) $3x^3 + 18x^2 - 4x - 24$

$$13) x^2 + 25$$

$$14) c^2 - 441$$

$$15) 196 - m^2$$

$$16) y^2 - 9$$

$$17) g^4 - 49$$

$$18) b^2 - 4$$

$$19) s^2 - 15$$

$$20) y^2 + 81$$

Review Section:

$$21) 3(2x + 4) = -6$$

$$22) 4x + 3 + 2x = 27$$

$$23) 3(4x - 5) + 2(11 - 2x) = 31$$

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GCF/DOTS

1B HW

1) $7x^5(2x^4 - x^2 + 3)$

2) $4x^4(8x^2 - 3x - 4)$

3) $8xy^2(2x^4 - x^3y + 3xy^2 - 4y^3)$

4) $2b^8(12b^3 + 2b^2 - 3b + 1)$

5) $11xy(x^2y^2 + 11xy - 8)$

6) $5a(15a^4 + 3a^3 - 5)$

7) $12a^3b^4c^3(11a^2 - 4ac + 6c^2)$

8) $16x^5 + 12xy - 9y^5$; *prime, not factorable*

9) $(x + 1)(3x - 4)$

10) $(x + 5)(x^2 + 2)$

11) $(x - 2)(7x + 1)$

12) $(x + 6)(3x^2 - 4)$

13) $x^2 + 25$; *prime, not factorable*

14) $(c + 21)(c - 21)$

15) $(14 + m)(14 - m)$

16) $(y + 3)(y - 3)$

17) $(g^2 + 7)(g^2 - 7)$

18) $(b + 2)(b - 2)$

19) $s^2 - 15$; *prime, not factorable*

20) $y^2 + 81$; *prime, not factorable*

21) $x = -3$

22) $x = 4$

23) $x = 3$

3. Case Two Factoring

Case Two trinomial factoring is used when the leading coefficient is not 1.

Example) Factor $2x^2 - 7x + 6$

Step 1 - List out a,b, and c
 $2x^2 - 7x + 6$
 $ax^2 + bx + c$

$a = 2$
 $b = -7$
 $c = 6$

Step 2 - Split the middle term

Step 3 - Determine the two middle term signs
 Look at the last sign
 Because (+) **S**um the signs are the **S**ame
 The signs are the same as the first sign (+)

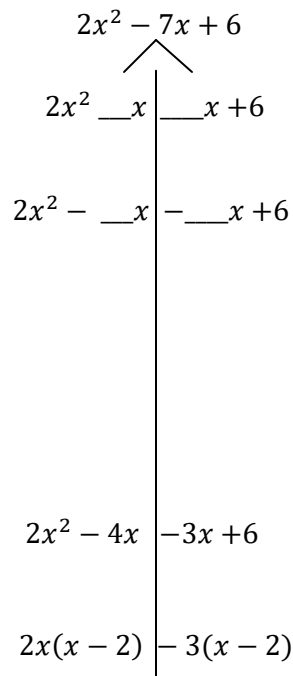
Step 4 - To figure out the coefficients needed
 multiply $(a \cdot c) = (2 \cdot 6) = 12$
 Therefore we will need factors of 12 with a sum of 7

Factors (a)(c)	Sum (b)
Factors 12	Sum 7
1, 12	13
2, 6	8
3, 4	7

****Values in table will always be positive****
****Always write your bigger number first****

Step 5 - Factor a GCF out of the created binomials

Step 6 - Factor out the common binomial to create a second binomial



$(x - 2)(2x - 3)$

Let's try an example:

1) $x^2 - 8x + 32$

Do we notice any shortcuts that we can use when it is a trinomial when $a = 1$?

Last year you learned that when it was a trinomial written in $ax^2 + bx + c$ form that you factor using a method called Case 2 or Grouping. Now that we have been practicing factoring for a while, there is a shortcut for a trinomial when $a = 1$. Let's look at what that shortcut is:

4. Trinomial: Case One Factoring

A **trinomial** is a polynomial that consists of 3 terms. Case One trinomial factoring is used when the leading coefficient is _____.

Factoring (Sum)

Ex) Factor: $x^2 - 18x + 32$

Step 1 - List out a,b, and c

$$ax^2 + bx + c$$

$$x^2 - 18x + 32$$

$$(x \quad)(x \quad)$$

$$a = 1 \quad b = -18 \quad c = 32$$

$$a = 1 \text{ (case 1 factoring)}$$

Step 2 - Create two binomial parentheses

Step 3 - Find the factors to place in the

Binomials (make a table)

Factors (a)(c)32	Sum (b)18
1, 32	33
2, 16	18
4, 8	12

Step 4 - Determine the signs for the binomials

$$(x - 16)(x - 2)$$

**Sum means Same signs

$$(x - 16)(x - 2)$$

The larger number gets the sign of b

The other must be the same sign

Practice: Factor each of the following trinomials in the space provided.

1) $x^2 - 8x + 12$

2) $x^2 + 10x + 16$

3) $x^2 - 17x + 52$

4) $x^2 - 9x + 20$

Factoring (Difference)

Ex) Factor: $x^2 + 4x - 32$

Step 1 - List out a,b, and c

$$ax^2 + bx + c$$

$$a = 1 \quad b = 4 \quad c = -32$$

Step 2 - Create two binomial parentheses

$$x^2 + 4x - 32$$

$$a = 1 \text{ (case 1 factoring)}$$

Step 3 - Find the factors to place in the

$$(x \quad)(x \quad)$$

$$c(-) = \text{difference}$$

Binomials (make a table)

$$(x - 8)(x - 4)$$

Factors (a)(c)32	Difference (b)4
1, 32	31
2, 16	14
4, 8	4

Step 4 - Determine the signs for the binomials

**Difference means Different signs

The larger number gets the sign of b

The other must be a different sign

$$(x + 8)(x - 4)$$

****Remember****

For Sum problems, signs are the Same.

For Difference problems, signs are Different!

Practice: Factor each of the following trinomials in the space provided.

1) $x^2 - 4x - 21$

2) $x^2 + 8x - 33$

3) $x^2 + 4x - 60$

4) $x^2 - x - 20$

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Factoring: Case Two into Case One 1C HW

Directions: Answer the following questions completely. Please make sure to show all work that was shown in class!

Factoring (Sum)

1) $x^2 + 16x + 64$

2) $x^2 - 12x + 27$

3) $x^2 - 22x + 121$

4) $x^2 + 17x + 72$

5) $x^2 + 11x + 30$

6) $x^2 + 15x + 44$

Factoring (Difference)

7) $x^2 - 3x - 54$

8) $x^2 + 10x - 56$

9) $x^2 + 10x - 24$

10) $x^2 + 16x - 36$

11) $x^2 - 5x - 6$

12) $x^2 - 11x - 26$

Review Section:

13) Solve for x:

$$7(4x - 5) - 4(6x + 5) = -91$$

14) Solve for x:

$$102 = 69 - 7x + 3x$$

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Factoring: Case Two into Case One

1C HW

1) $(x + 8)(x + 8)$

2) $(x - 9)(x - 3)$

3) $(x - 11)(x - 11)$

4) $(x + 9)(x + 8)$

5) $(x + 6)(x + 5)$

6) $(x + 11)(x + 4)$

7) $(x - 9)(x + 6)$

8) $(x + 14)(x - 4)$

9) $(x + 12)(x - 2)$

10) $(x + 18)(x - 2)$

11) $(x - 6)(x + 1)$

12) $(x - 13)(x + 2)$

13) $x = -9$

14) $x = -8.25$

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Factoring: Case One 1D

As we discussed yesterday, there is a shortcut for factoring a trinomial when $a = 1$. Please make note that you can still factor using grouping if that is more comfortable for you.

Today we are going to practice Case One Factoring where both sum and difference problems are mixed together:

Let's try some!

1) $x^2 + 13x - 30$

2) $x^2 + 15x + 56$

3) $x^2 - 5x - 24$

4) $x^2 + 5x - 36$

5) $x^2 - 15x + 54$

6) $x^2 - 10x + 16$

$$7) x^2 - 8x - 20$$

$$8) x^2 + 14x + 49$$

$$9) x^2 + 6x + 5$$

$$10) x^2 - x - 20$$

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Factoring: Case One 1D HW

Directions: Answer the following questions completely. Please make sure to show all work that was shown in class!

1) $x^2 + 4x - 32$

2) $x^2 + 3x + 2$

3) $x^2 + 4x + 3$

4) $x^2 + x - 12$

5) $x^2 - 7x - 18$

6) $n^2 - 7n - 120$

7) $h^2 + 15h + 44$

8) $x^2 - 22x + 40$

Review Section: Factor the following completely.

9) $x^2 - 16$

10) $36x^2 - 25$



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"Just a darn minute — yesterday
you said that X equals two!"

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Factoring: Case One

1D HW

1) $(x + 8)(x - 4)$

2) $(x + 2)(x + 1)$

3) $(x + 3)(x + 1)$

4) $(x + 4)(x - 3)$

5) $(x - 9)(x + 2)$

6) $(n - 15)(n + 8)$

7) $(h + 11)(h + 4)$

8) $(x - 20)(x - 2)$

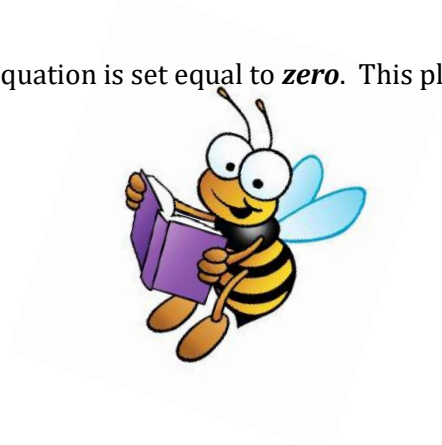
9) $(x + 4)(x - 4)$

10) $(6x + 5)(6x - 5)$

Solving Quadratic Equations

Steps for Solving Quadratic Equations:

- 1) Move all terms to the same side of the equal sign, so the equation is set equal to **zero**. This places the equation in standard form: $ax^2 + bx + c = 0$
- 2) **Factor** the algebraic expression.
- 3) Set each factor equal to **zero**.
- 4) **Solve** each resulting equation.
"T it up"



Examples:

1) Solve for x: $x^2 - x - 6 = 0$

2) Solve for c: $c^2 - c = 12$

3) Solve for y: $2y^2 = 32$

4) Solve for x: $\frac{x}{9} = \frac{144}{x}$

5) Solve for x: $2y^2 + 4 = 9y$



6) Solve for x: $x^2 - 3x = 10$

7) Solve for x: $\frac{x+2}{2x+1} = \frac{x-2}{3}$

Directions: Answer the following questions completely. Make sure to *SHOW ALL WORK* or you will not get credit.



Example: Solve for x : $x^2 - x = 6$

$$\begin{array}{r} x^2 - x = 6 \\ -6 \quad -6 \\ \hline x^2 - x - 6 = 0 \end{array}$$

$$(x - 3)(x + 2) = 0$$

$$\begin{array}{l|l} (x-3)(x+2) = 0 & \\ \hline x-3=0 & x+2=0 \\ +3 \quad +3 & -2 \quad -2 \\ \hline x=3 & x=-2 \end{array}$$

$$x = \{-2, 3\}$$

Steps to follow:

- 1) Move all terms to the same side of the equal sign, so that the equation is set equal to 0 and the x^2 term is positive.
- 2) Factor the algebraic expression.
- 3) "T" off the equation, and then set each factor equal to zero.
- 4) Solve each of the resulting equations.
- 5) $x = 3$ and $x = -2$ are the roots of the equation $x^2 - x - 6 = 0$.

1) $x^2 + 12x + 32 = 0$

2) $m^2 + 13m = 30$

$$3) a^2 + 25 = 10a$$

$$4) 2d^2 = 18$$

$$5) b^2 - 5b = 36$$

$$6) x^2 = 16$$

$$7) 2x^2 + 14x = -12$$

$$8) \frac{x}{7} = \frac{4}{x-3}$$

$$9) \frac{8}{x} = \frac{x+2}{3}$$

Review Section:

10) Solve for the following value of x: $-13 = 3(-2x + 1) + 8$

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Solving Quadratics

1E HW

1) $x = \{-8, -4\}$

2) $m = \{-15, 2\}$

3) $a = \{5\}$

4) $d = \{-3, 3\}$

5) $b = \{-4, 9\}$

6) $x = \{-4, 4\}$

7) $x = \{-6, -1\}$

8) $x = \{-4, 7\}$

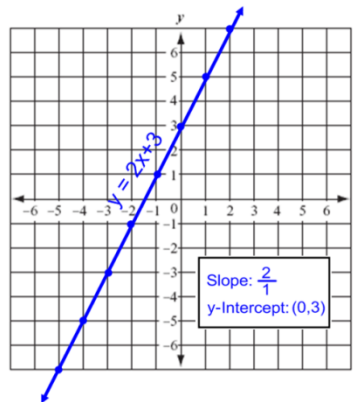
9) $x = \{-6, 4\}$

10) $x = 4$

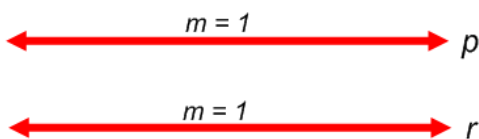
Parallel and Perpendicular Lines

Recall:

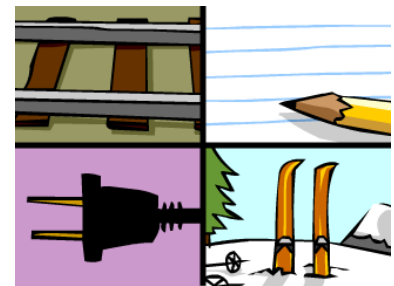
Equation of a Line: $y = mx + b$
 Slope: m
 y-intercept: b



•**Parallel Lines:** Lines that *never* intersect. These lines have the *same* slope.

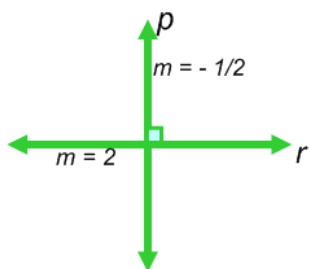


$$p \parallel r$$

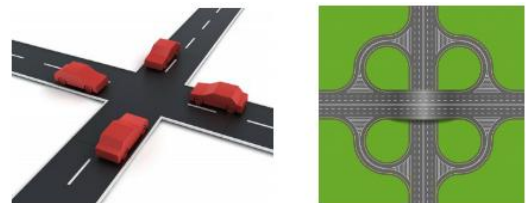


Example: $y = 2x + 4$ is parallel to $y = 2x - 7$

•**Perpendicular Lines:** Lines that intersect to form *right angles*. These lines have *negative reciprocal* slopes.



$$p \perp r$$



Example: $y = 2x + 4$ is perpendicular to $y = -\frac{1}{2}x - 7$

Examples:

1) What is the slope of the line that is parallel to $y = 3x + 5$?

2) What is the slope of the line that is perpendicular to $y = -6x - 12$?

3) What is the slope of the line that is parallel to $4y - 3x = -12$?

4) What is the slope of the line that is perpendicular to $y = -\frac{1}{3}x - 9$?

5) What is the slope of the line that is perpendicular to $2y + 6x = 42$?

Equation of a Line: Point-Slope Form

Recall: An equation of a line is $y = mx + b$. This is the form that you learned last year. Where:

m = slope

b = y-intercept

Point-Slope Form: A linear equation can also be written in point-slope form. This form is helpful when you are given a *point* and a *slope*.

$$y - y_1 = m(x - x_1)$$

Where:

m = slope

(x, y) = any point on the line

You must know how to do these questions! You will see them on almost every homework assignment!

Examples:

1) Write an equation that is parallel to the line $y = 2x + 3$ and passes through the point $(-4,5)$.

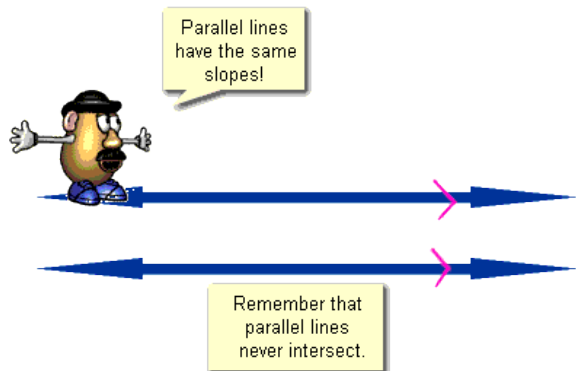
2) Write an equation that is perpendicular to the line $y = 2x - 7$ and passes through the point $(-8,12)$.

3) Write the equation of the line that passes through the point $(-3,2)$ and is perpendicular to $y = -\frac{1}{2}x + 5$.

4) What is the equation of the line that passes through the point $(-2,-1)$ and is parallel to $2y - 4x = 9$?

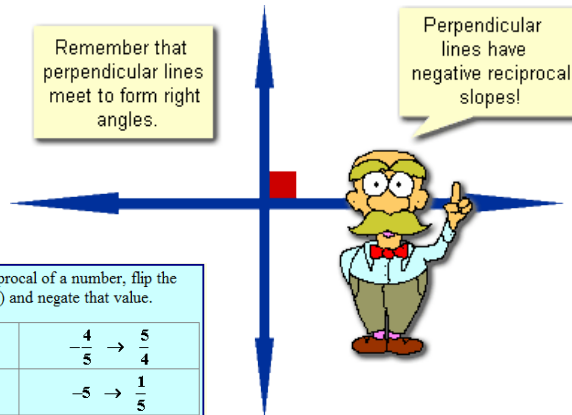
5) What is the equation of the line that passes through the point $(4,6)$ and is perpendicular to $3y - 6x = -12$?

Directions: Answer the following questions completely. Make sure to show all work that was shown in class. These are all questions you will see on your Common Core exam. ☺



To find a negative reciprocal of a number, flip the number over (invert) and negate that value.

$\frac{1}{2} \rightarrow -\frac{2}{1} = -2$	$-\frac{4}{5} \rightarrow \frac{5}{4}$
$3 = \frac{3}{1} \rightarrow -\frac{1}{3}$	$-5 \rightarrow \frac{1}{5}$



1) What is the slope of a line perpendicular to the line whose equation is $y = -\frac{2}{3}x - 5$?

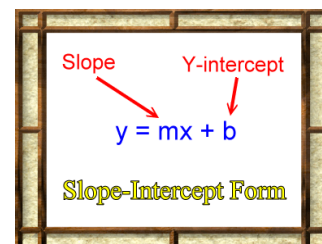
(1) $-\frac{3}{2}$

(2) $-\frac{2}{3}$

(3) $\frac{2}{3}$

(4) $\frac{3}{2}$

2) Find the slope of a line parallel to the line whose equation is $2y - 6x = 4$.



3) What is the equation of a line that is parallel to the line whose equation is $y = x + 2$?

(1) $x + y = 5$

(2) $2x + y = -2$

(3) $y - x = -1$

(4) $y - 2x = 3$

8) What is an equation of the line that passes through the point $(-2,3)$ and is parallel to the line whose equation is $y = \frac{3}{2}x - 4$?

(1) $y = \frac{-2}{3}x$

(2) $y = \frac{-2}{3}x + \frac{5}{3}$

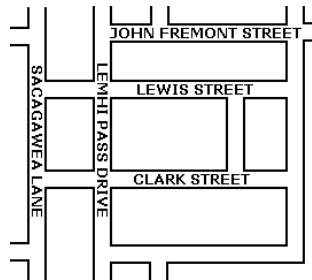
(3) $y = \frac{3}{2}x$

(4) $y = \frac{3}{2}x + 6$

9) Given the diagram to the right, answer the following questions:

(a) Name two streets that are parallel:

(b) Name two streets that are perpendicular:



Review Section:

10) Solve the following algebraically for all values of x .

$$x^2 - 4x = 32$$

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Parallel/Perpendicular Lines 1F HW

Geometry

1) (4)

2) $\parallel m = 3$

3) (3)

4) $\perp m = \frac{4}{3}$

5) $\parallel m = 2$

6) (2)

7) (2)

8) (4)

9) (a) Clark Street and Lewis Street
(b) Clark Street and Lemhi Pass Drive

10) $x = \{-4, 8\}$