

# Projectile Motion

**Two Dimensional Motion of balls,  
canon balls, arrows**

# What would you like to know?

1. How far is it going? (“range” or “horizontal displacement”)
2. How high is it going? (“apex” or “maximum height”)
3. How long is it in the air? (“time of flight” or “TOF”)
4. How long until it reaches its apex? (“Time to apex” or “apex time”)

# Steps for Projectile Motion

- 1. Resolve initial Velocity into  $V_x$  and  $V_y$ .  $V_x = V \cos(\text{angle})$  and  $V_y = V \sin(\text{angle})$ .
- 2. Time of flight =  $(2 \times V_y) / 9.8$  (seconds)
- 3. Range =  $V_x \times \text{Time of flight}$  (meters)
- 4. Time to apex =  $\frac{1}{2} \times \text{Time of flight}$  (seconds)
- 5. Apex =  $(V_y \times \text{Time to apex}) + (\frac{1}{2} \times -9.8) \times (\text{Time to apex})^2$  (meters)

# Example

- Thomas Cani throws a pass to Chuck Myers at 20 m/s at 30 degrees above the horizontal.
- Determine the TOF, range, T(apex), and apex.
- 1.  $V_x = 20\cos(30) = 17.3 \text{ m/s}$
- $V_y = 20\sin(30) = 10 \text{ m/s}$
- 2.  $\text{TOF} = (2 \times V_y)/9.8 = 2 \text{ seconds}$
- 3.  $T(\text{apex}) = \frac{1}{2} \times 2 \text{ seconds} = 1 \text{ second}$
- 4.  $\text{Range} = V_x \times \text{TOF} = 34.6 \text{ meters}$
- 5.  $\text{Apex} = V_y \times T(\text{apex}) + \frac{1}{2} \times -9.8 \times T(\text{apex})^2 =$
- $10 - 4.9 = 5.1 \text{ meters}$

# Group Activity

- Determine the TOF, range, and Time(apex) for the following projectiles:
  - 1. 35 m/s at 15 degrees above horizontal
  - 2. 35 m/s at 25 degrees above horizontal
  - 3. 35 m/s at 35 degrees above horizontal
  - 4. 35 m/s at 45 degrees above horizontal
  - 5. 35 m/s at 55 degrees above horizontal

# Problem #1

- 1.  $V_x = 33.8 \text{ m/s}$      $V_y = 9.06 \text{ m/s}$
- 2. TOF = 1.85 s
- 3. Time(apex) = 0.92 s
- 4. Range =  $33.8 \times 1.85 = 62.53 \text{ m}$

# Problem #2

- 1.  $V_x = 31.7 \text{ m/s}$        $V_y = 14.8 \text{ m/s}$
- 2. TOF = 3 s
- 3. Time(apex) = 1.5 s
- 4. Range =  $31.7 \times 3 = 95.1 \text{ m}$

# Problem #3

- 1.  $V_x = 28.67 \text{ m/s}$      $V_y = 20 \text{ m/s}$
- 2. TOF = 4.1 s
- 3. Time to apex = 2.05 s
- 4. Range =  $28.67 \times 4.1 = 117.5 \text{ m}$



# Problem #4

- 1.  $V_x = 24.75 \text{ m/s}$      $V_y = 24.75 \text{ m/s}$  (equal !)
- 2. TOF = 5.05 s
- 3. Time to apex = 2.53 s
- 4. Range =  $V_x \times \text{TOF} = 24.75 \times 5.05 = 125 \text{ m}$

# Problem #5

- 1.  $V_x = 20.08 \text{ m/s}$      $V_y = 28.67 \text{ m/s}$
- 2. TOF = 5.85 s
- 3. Time to apex = 2.93 s
- 4. Range =  $20.08 \times 5.85 = 117.47 \text{ m}$

# Group Activity

- Determine the apex for the previous five problems.
- 1. 35 m/s at 15 degrees
- 2. 35 m/s at 25 degrees
- 3. 35 m/s at 35 degrees
- 4. 35 m/s at 45 degrees
- 5. 45 m/s at 55 degrees

$$\text{Apex} = V_y \times T(\text{apex}) + \frac{1}{2} (-9.8)T(\text{apex})^2$$

- 1. apex = 8.34 – 4.15 = 4.19 m
- 2. apex = 22.2 – 11.02 = 11.18 m
- 3. apex = 41 – 20.6 = 20.4 m
- 4. apex = 62.6 – 31.36 = 31.24 m
- 5. apex = 84.0 – 42.0 = 42 m