Name: ______________________________

Skill Sheet 17-B Ratios and Proportions

Speed, the density of a solid, and pressure are all measurements that are given as ratios. When you set two ratios equal to each other we call this equation a proportion. In this skill sheet, you will investigate the techniques used to analyze and manipulate ratio- and proportion-based formulas and examine a few specific examples of how these concepts are applied in science.

1. Understanding ratios and proportions

What is a *ratio*? Ratios are expressions of relationships or comparisons. In general, ratios express the relationship:

$$\frac{\text{amount or magnitude of a sample}}{\text{total amount or magnitude of system containing the sample}}$$

For example, suppose you have a jar filled with 100 marbles. This is the total number of marbles in the system. You are asked to report the number of red marbles in the jar. This is the amount of a specific sample of the marbles. After counting, you find that 25 of the 100 marbles are red. As a relationship, you could say that you had 25 marbles compared to the total 100 marbles in the jar. You can express this relationship—25 red marbles compared to 100 total marbles—as a fraction:

$$\frac{\text{25 red marbles}}{\text{100 total marbles}}$$

You can use this ratio to analyze other systems of marbles.

Let's say this particular brand of marbles come in jars of 100 or 300. Because you know how many marbles out of 100 are red, you can predict how many marbles out of 300 are red by setting up a *proportion*.

The advantage of setting up a proportion is that you can get a good estimate of the number of something without having to spend time counting each marble. The proportion for the marbles is set up for you below:

$$\frac{\text{25 red marbles}}{\text{100 total marbles}} = \frac{\text{? red marbles}}{\text{300 total marbles}}$$

Now that you have the proportion, you need to solve it. To solve a proportion, you use a technique called *cross multiplication*.

2. Cross Multiplication

Cross multiplication is a mathematical technique to solve proportions, also known as *equivalent fractions*. Our sample problem is an example of equivalent fractions. Each fraction represents a system. One system has 100 marbles and the other has 300 marbles. However, each of these systems share the same relationship—that 25 marbles in every 100 marbles are red.

When using cross multiplication, you multiply across the equal sign between the fractions. You first multiply the numerator of the first fraction by the denominator of the second fraction:

$$\frac{\text{25 red marbles}}{\text{100 total marbles}} = \frac{\text{? red marbles}}{\text{300 total marbles}}$$

You then multiply the denominator of the first fraction by the numerator of the second fraction:

$$\frac{\text{25 red marbles}}{\text{100 total marbles}} = \frac{\text{? red marbles}}{\text{300 total marbles}}$$

This gives you a new formula:

$$(\text{25 red marbles}) \times (\text{300 total marbles}) = (\text{100 total marbles}) \times (\text{? red marbles})$$

The next step in cross multiplication is to solve for the number of red marbles in the jar containing 300 marbles.

For this problem, you divide each side of the equation by 100 total marbles. On the right hand side of the equation, you have 100 total marbles divided by 100 total marbles. These values cancel out leaving "**? red marbles**."

$$\frac{(\text{25 red marbles}) \times (\text{300 total marbles})}{\text{100 total marbles}} = \frac{(\text{100 total marbles}) \times (\text{? red marbles})}{\text{100 total marbles}}$$

$$\text{75 red marbles} = \text{? red marbles}$$

Solving the left-hand side of the equation results in a value of 75 red marbles. Based on the relationship for the smaller jar of marbles, this value is a good prediction of how many red marbles you would find in the jar of 300 marbles.

3. Examples of ratios and proportions

Example 1: Density

Density is a relationship between the mass of a substance and the amount of space it occupies:

$$\text{Density} = \frac{\text{Mass}}{\text{Volume}}$$

For a single type of substance, the density relationship is constant. Therefore, you can use this relationship, or ratio, to predict the mass or volume for a sample of the substance using equivalent fractions:

$$\text{Density}_1 = \text{Density}_2$$

$$\frac{M_1}{V_1} = \frac{M_2}{V_2}$$

where M_1 and V_1 are the mass and volume of a *sample* of a substance and M_2 and V_2 are the mass and volume of the substance. Here's an example problem:

You have a block of aluminum with a mass of 369 g and a volume of 136.4 cm^3. If the block is cut in half, what is the mass of the resulting sample?

You have reduced the volume of the block by half, so the new volume is 68.2 m^3. The density of the sample and the relationship between the mass and volume is constant, so you can set up a set of equivalent fractions:

$$\frac{M_1}{V_1} = \frac{M_2}{V_2}$$

$$\frac{369\ \text{g}}{136.4\ \text{cm}^3} = \frac{M_2}{68.2\ \text{cm}^3}$$

$$(369\ \text{g}) \times (68.2\ \text{cm}^3) = (136.4\ \text{cm}^3) \times (M_2)$$

$$\frac{(369\ \text{g}) \times (68.2\ \text{cm}^3)}{(136.4\ \text{cm}^3)} = \frac{(136.4\ \text{cm}^3) \times (M_2)}{(136.4\ \text{cm}^3)}$$

$$185\ \text{g} = M_2$$

The new block will have a mass of 185 g. If you calculate the densities of the two blocks, the original and the new, they both have the same value 2.7 g/cm^3.

Example 2: Pressure

Pressure is defined as a force acting over a given area. Mathematically, you may express pressure with the formula:

$$\text{Pressure} = \frac{\text{Force}}{\text{Area}}$$

Here's an example problem:

Consider a machine designed to produce 4 N/cm^2 of pressure during a manufacturing process. How much area would be required deliver a force of 36 N?

Knowing that the relationship of force to area in this case is constant, you can use proportions and ratios to evaluate the area over which the machine needs to apply this new force. Setting up the equivalent fractions:

$$\frac{F_1}{A_1} = \frac{F_2}{A_2}$$

$$\frac{4\text{ N}}{1\text{ cm}^2} = \frac{36\text{ N}}{A_2}$$

$$(4\text{ N}) \times (A_2) = (1\text{ cm}^2) \times (36\text{ N})$$

$$\frac{(4\text{ N}) \times (A_2)}{(4\text{ N})} = \frac{(1\text{ cm}^2) \times (36\text{ N})}{(4\text{ N})}$$

$$A_2 = 9\text{ cm}^2$$

4. Working with ratios and proportions

1. A barrel contains 250 apples. 100 of the apples are red, and 150 of the apples are green. Express the number of red and green apples to the total number in the barrel as two ratios.

2. The number of jazz and blues CD's to the total number of CD's at a music store can be expressed with the ratio $^1/_4$. If there are 1,000 total CD's in the store, how many belong in the jazz and blues category?

3. The owner of the music store in question (2) sees that jazz and blues CD's are selling particularly well. He changes the number of the jazz and blues CD's that he stocks. He now carries 500 jazz and blues CD's out of his total stock of 1,000 CD's. What ratio expresses this new amount of jazz and blues CD's to total CD's in the store?

4. A sample of material has a mass of 15 g and occupies a space of 45 cm^3. If material is added to the sample so that the new mass equals 60 g, how much space will the sample now occupy?

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5. A woman needs to ship a 5 m^2 glass block to an artist in California. She knows that a 2 m^2 glass block has a mass of 6 kg. To give her customer a prediction of the shipping cost, she needs to know the mass of the 5 m^2 block. What mass value would she tell the post office to calculate the cost of shipping?

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6. The concept of power expresses the rate at which work is performed. It is calculated using the equation:

$$\text{Power} = \frac{\text{work}}{\text{time}}$$

A machine is capable of producing 250.0 joules of work in 45.00 seconds. If the machine operates for 600.0 seconds, how much work will be performed?

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7. A baker has to bring cheesecakes to a big Hollywood party. Each cheesecake will serve 12 guests. The total number of guests expected at the party is 720.

a. How many cheesecakes will the baker need to prepare for the party?

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b. The host of the party decides that he wants $^1/_4$ of the cheesecakes to be blueberry, $^1/_4$ of the $^1/_4$ cheesecakes to be chocolate, and $^1/_2$ of the cheesecakes to be plain. How many blueberry, chocolate, and plain cheesecakes does the baker need to prepare?

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c. While talking to potential guests, the party host finds that about $^1/_3$ of them might prefer carrot cake to the cheesecake he was planning to offer. He, therefore, instructs the baker to prepare carrot cakes in addition to the cheesecakes. If the baker plans for $^1/_3$ of the guests having the option of carrot cake, how many carrot cakes does he need to bake (each carrot cake also serves 12 people)?
