Name: ________________________________ Date: ________________ Period: ______

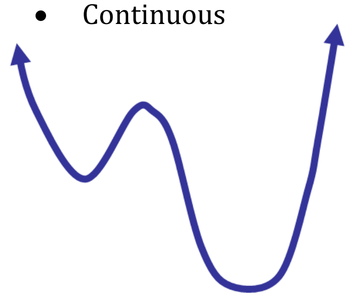
Chapter 2: Polynomial and Rational Functions
Topic 3: Polynomial Functions and their graphs

What does/doesn't a polynomial function graph look like?

Polynomial functions of any degree (linear, quadratic, or higher-degree) must have graphs that are smooth and continuous. There can be no sharp corners on the graph. There can be no breaks in the graph; you should be able to sketch the entire graph without picking up your pencil.

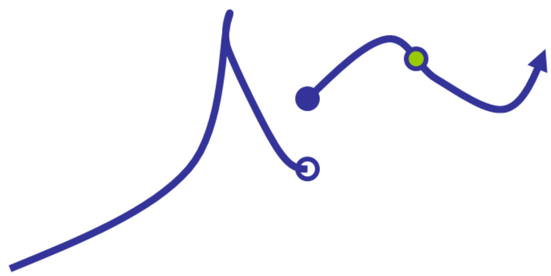
Polynomial function

- Smooth, rounded turns
- Continuous



Not a polynomial function

- Sharp Turn
- Discontinuous



End Behavior

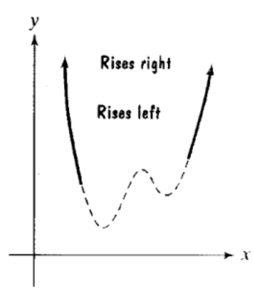
The behavior of the graph of a function to the far left or far right is called the end behavior. Although the graph of a polynomial function may have intervals where it increases or decreases, the graph will eventually continue to positive or negative infinity on both ends, without bound, as it rises or falls.

General Guidelines:

<u>When the highest power is EVEN:</u>

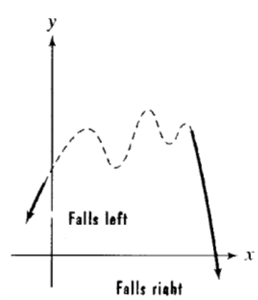
With a <u>positive</u> coefficient:

If the leading coefficient is positive, the graph rises to the left and to the right.



With a <u>negative</u> coefficient:

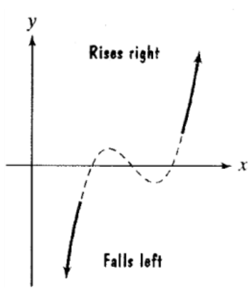
If the leading coefficient is negative, the graph falls to the left and to the right.



<u>When the highest power is ODD:</u>

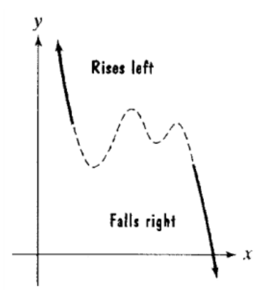
With a <u>positive</u> coefficient:

If the leading coefficient is positive, the graph falls to the left and rises to the right.



With a <u>negative</u> coefficient:

If the leading coefficient is negative, the graph rises to the left and falls to the right.



Zeros of a Polynomial Function: $f(x) = 0$

Recall: The highest degree of the equation will indicate how many roots the equation has.

Set the equation equal to zero and solve by factoring. These are the point where the graph interacts with the x-axis. Typically, the graph goes directly through the x-axis at these roots:

Example: Find all zeros of the function $f(x) = x^3 + 2x^2 - 9x - 18$

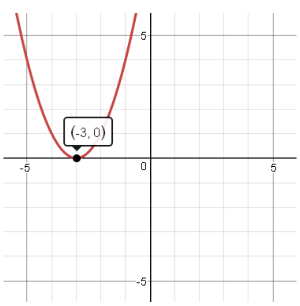
Example: Find all zeros of the function $f(x) = x^3 - 2x^2$

Multiplicity - the number of times a root is associated with an equation. When a root has a multiplicity (more than one) the graph CURVES at the x-axis

All of the examples below have a single root at -3. How many times does the root repeat for each?

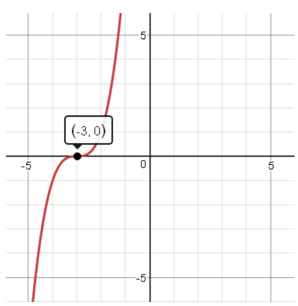
$f(x) = (x+3)^2$

$f(x) = (x+3)(x+3)$



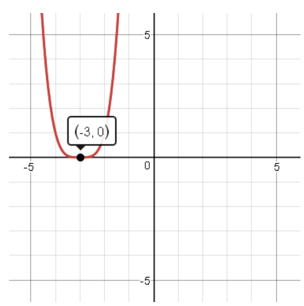
$f(x) = (x+3)^3$

$f(x) = (x+3)(x+3)(x+3)$



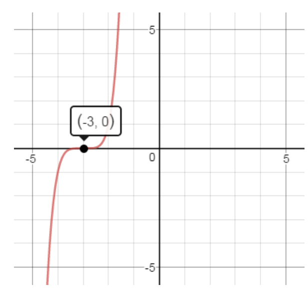
$f(x) = (x+3)^4$

$f(x) = (x+3)(x+3)(x+3)(x+3)$



$f(x) = (x+3)^5$

$f(x) = (x+3)(x+3)(x+3)(x+3)(x+3)$



<u>Make a conclusion:</u>

When a root is repeated an ODD number of times:

When a root is repeated an EVEN number of times:

Turning Points:

As a general rule, a polynomial with highest degree *n* will have *n-1* turning points on its graph.

This rule does not hold true when there is multiplicity.

Graphing Polynomial Functions

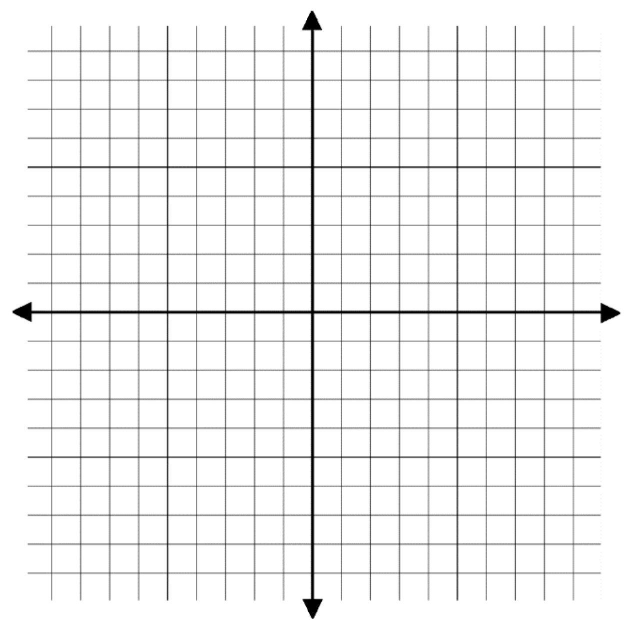
To graph polynomial functions we look for 4 key features:

1. **End Behavior**
   - An ODD highest power: Ends are in opposite directions
     - With a positive coefficient: Rise Right, Fall Left
     - With a negative coefficient: Rise Left, Fall Right
   - An EVEN highest power: Ends are in the same direction
     - With a positive coefficient: Rises
     - With a negative coefficient: Falls
2. **What are the x-intercepts?**
   - Solve by setting the equation equal to zero ($f(x) = 0$)
     - Repeated roots: The graph touches the x-axis and turns around
3. **What is the y-intercept?**
   - Solve by evaluating at zero ($f(0) =\ $)
4. **Turning Points**
   - The max number of turning points is one-less-than the highest power.

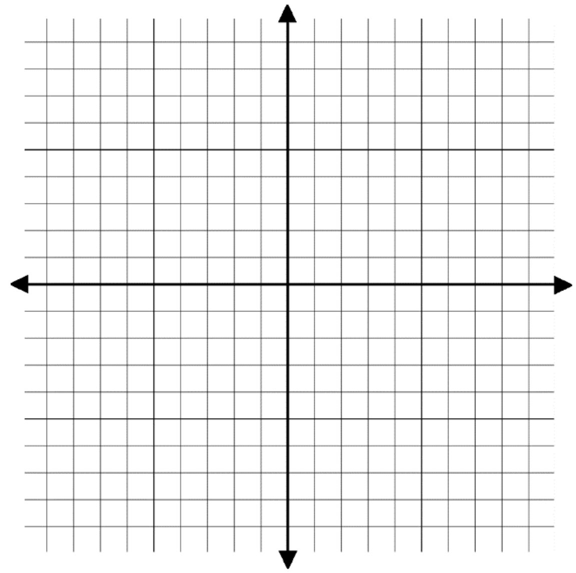
Then, plot all of the key points, and sketch a smooth polynomial.

Examples:

1. $f(x) = x^3 + 3x^2 - x - 3$



2. $f(x) = x^4 - 2x^2 + 1$



3. $f(x) = x^3 - 3x^2$