

Name: \_\_\_\_\_ Date: \_\_\_\_\_ Period: \_\_\_\_\_

## Sequences - Packet #1

**Definition:** A sequence is a set of numbers in a specific order. 2, 5, 8 ... is an example of a sequence.

Note that a sequence may have either a finite or an infinite number of terms.

The terms of a sequence are the individual numbers in the sequence. If we let  $a_1$  represent the first term of a sequence,  $a_n$  represent the  $n^{\text{th}}$  term, and  $n$  represent the term number, then the sequence is represented by  $a_1, a_2, a_3 \dots a_n \dots$ . In the example above,  $a_1=2, a_2=5, a_3=8$ , etc.

### Arithmetic Sequences

**Definition:** An arithmetic sequence is a sequence in which each term, after the first, is formed by adding the preceding term to a common difference.

An arithmetic sequence can be represented by  $a_1, a_1 + d, a_2 + d, a_3 + d \dots$   
where  $a_1$  is the initial term, and  $d$  is the common difference

- In the sequence 2, 5, 8, 11... the initial term ( $a_1$ ) is 2 and the common difference ( $d$ ) is 3.
- In the sequence 1, 3, 5, 7... the initial term ( $a_1$ ) is 1 and the common difference ( $d$ ) is 2
- In the sequence 2, 8, 14, 20... the initial term ( $a_1$ ) is 2 and the common difference ( $d$ ) is 6.

Each has the property that the difference between any two immediately successive terms is constant.

The existence of a common difference is the characteristic feature of an arithmetic sequence. To test whether a given sequence is an arithmetic sequence, determine whether a common difference exists between every pair of successive terms.

- For example, 4, 8, 16, 32... is not an arithmetic sequence because the difference between the first two terms is 4, but the difference between the second and third terms is 8.

### Exercises:

A. Write the first five terms of the arithmetic sequence in which  $a_1$  and  $d$  are given as follows.

1.)  $a_1 = 17, d = 12$                       *17, 29, 41, 53, 65...*

2.)  $a_1 = 3, d = \frac{3}{2}$

3.)  $a_1 = -6, d = -3$

B. Write the last four terms before  $a_n$  of the arithmetic sequence in which  $a_n$  and  $d$  are as follows.

4.)  $a_n = -7, d = 6$                       *... -31, -25, -19, -13, -7*

5.)  $a_n = 36, d = -5$

6.)  $a_n = 5, d = 10$

C. Tell whether each of the following is an arithmetic sequence. In those sequences which are arithmetic sequences, find the common difference and write the next two terms.

7.) -2, 3, 8...                      *Yes, the common difference is 5. ... 13, 18...*

8.) 5, -1, -7...

9.)  $-9x, -2x, 5x...$

## ALL WORK (NEATLY ORGANIZED) IN A NOTEBOOK

If  $a_1$  is the first term of an arithmetic sequence,  $a_n$  the  $n^{\text{th}}$  term,  $d$  is the common difference, a formula for finding the value of the  $n^{\text{th}}$  term of an arithmetic sequence is:

$$a_n = a_1 + (n - 1)d \quad \text{*not given on the regents exam reference sheet}$$

The formula for the  $n^{\text{th}}$  term of an arithmetic sequence may be used to find any term of the sequence. This is done by choosing the appropriate value of  $n$  and substituting in the formula above.

### Model Solutions to Common Questions using this formula

#### Finding $n^{\text{th}}$ term given a piece of the sequence

Ex: Find the 13<sup>th</sup> term of 2, 8, 14, 20, 26...

- State the formula  $a_n = a_1 + (n - 1)d$
- List known & unknown values  $a_n = a_{13}, a_1 = 2, n = 13, d = 6$
- Substitute into the formula  $a_{13} = 2 + (13 - 1)6$
- Solve  $a_{13} = 74$

#### Identify the arithmetic sequence given two terms

Ex: Write the infinite arithmetic sequence whose first term is 5 and whose 7<sup>th</sup> term is 17

- State the formula  $a_n = a_1 + (n - 1)d$
- List known & unknown values  $a_n = 17, a_1 = 5, n = 7, d = ?$
- Substitute into the formula  $17 = 5 + (7 - 1)d$
- Solve  $17 = 5 + 6d$   
 $2 = d$
- Apply the solution as a series 5, 7, 9, 11, 13, 15, 17....  
*Hey, look at that.. the 7<sup>th</sup> term is 17!*

#### Identify the position of a value

Ex: Which term of 14, 21, 28, ..... is 112?

- State the formula  $a_n = a_1 + (n - 1)d$
- List known & unknown values  $a_n = 112, a_1 = 14, n = ?, d = 7$
- Substitute into the formula  $112 = 14 + (n - 1)7$
- Solve  $112 = 14 + 7n - 7$   
 $15 = n$
- Apply the solution 112 is the 15<sup>th</sup> term

### Exercises:

D. Find the  $n^{\text{th}}$  term of the arithmetic sequence in which:

10.)  $a_1 = 11, d = -2, n = 19$

11.)  $a_1 = 1.5, d = 0.5, n = 16$

E. Find the term indicated in each of the following sequences.

12.) 43<sup>rd</sup> term of -19, -15, -11, .....

13.) 58<sup>th</sup> term of 10, 4, -2, .....

14.) 13<sup>th</sup> term of 8, 13, 18, .....

F. Answer each of the following.

15.) Which term ( $n$ ) of 14, 21, 28, ..... is 112?

16.) Which term of 3, -2, -7, ..... is -57?

17.) Which term of 23, 30, 37, ..... is 240?

18.) Find the common difference ( $d$ ) in the arithmetic sequence whose 1<sup>st</sup> term is 4 and whose 11<sup>th</sup> term is 64.

19.) How many terms ( $n$ ) are there in the sequence  $-13, -8, -3, 2, \dots, 37$ ?

20.) How many terms are there in the sequence 9, 33, 57, 81, ..., 633?

**Arithmetic means**

**Definition:** Arithmetic means are the terms between any two other terms of an arithmetic sequence.

In the sequence 1, 3, 5, 7, 9, 11, 13, ..., the terms 5, 7, and 9 are called arithmetic means between 3 and 11.

**Definition:** A single arithmetic mean between two numbers is what is commonly called the average of the two numbers. It is equal to  $\frac{1}{2}$  the sum of the two numbers.

**Model Solution**

Ex: Insert 3 arithmetic means between 7 and -9.

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|--|--|
| ▪ Write the sequence, leaving blank spaces for the missing means                         | 7, __, __, __, -9                            |
| ▪ Determine the values of $a_1$ , $a_n$ , and $n$  | $a_1 = 7$ , $a_5 = -9$ , $n = 5$             |
| ▪ Substitute in the formula for $a_1$ , $a_n$ , $n$ and solve for $d$ .                  | $-9 = 7 + (5 - 1)d$<br>$d = -4$              |
| ▪ Write the sequence by adding the value of $d$ to each term to determine the next term. | 7, <u>3</u> , <u>-1</u> , <u>-5</u> , -9     |
| ▪ Answer the question appropriately  | The three arithmetic means are 3, -1, and -5 |

**Exercises:**

G. In each of the following, insert the indicated number of arithmetic means between the 2 given numbers.

21.) -4 and 5, 2 means

22.) 12 and 21, 2 means

23.) -6 and 24, 4 means

24.) 36 and 48, 3 means