Chapter 10A: Polynomials and their Graphs

Topic 1: Symmetry and End Behavior of Graphs

Topic 2: Graphing Polynomials (in factored form)

Topic 3: Graphing Polynomials (not in factored form)

Topic 4: Long Division of Polynomials

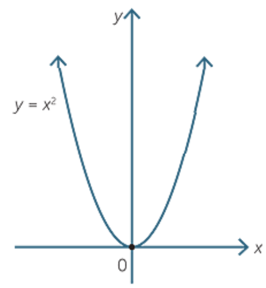
Topic 5: Remainder Theorem

Topic 1: Symmetry and End Behavior

Odd & Even Functions: by Symmetry

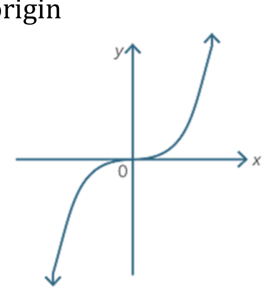
Functions are considered EVEN if they are symmetrical about the y-axis

Example: $y = x^2$



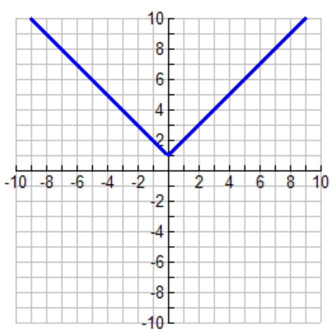
Functions are considered ODD if they are symmetrical about the origin

Example: $y = x^3$



Examples: Determine if the following functions are odd, even, or neither based on their graph. Explain your thinking.

1.



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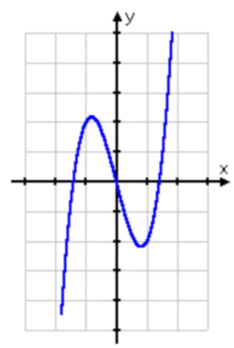

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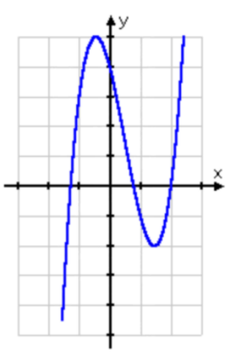

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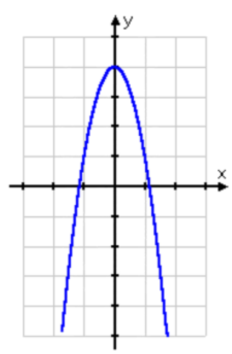


Odd & Even Functions: Algebraically

Definitions:

A function is EVEN if $\boldsymbol{f(-x) = f(x)}$

The equation does not change if x is replaced with $(-x)$

A function is ODD if $\boldsymbol{f(-x) = -f(x)}$

Every term in the equation changes sign when replaced with $(-x)$

If NEITHER of these statements is true, then the function is neither odd nor even.

Examples: Determine if the following functions are odd, even, or neither algebraically.

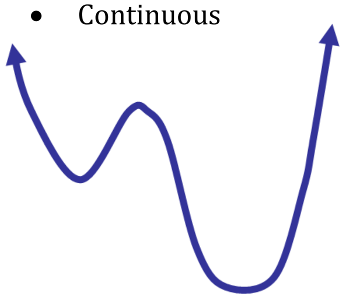
1. $f(x) = x^4 - 2x^2$
2. $g(x) = 2x^2 + 2x + 1$
3. $f(x) = x^3$
4. $h(x) = 4x^5 + 1$
5. $g(x) = \frac{3}{x^2+4}$
6. $f(x) = 7x^3 - x$

What does/doesn't a polynomial function graph look like?

Polynomial functions of any degree (linear, quadratic, or higher-degree) must have graphs that are smooth and continuous. There can be no sharp corners on the graph. There can be no breaks in the graph; you should be able to sketch the entire graph without picking up your pencil.

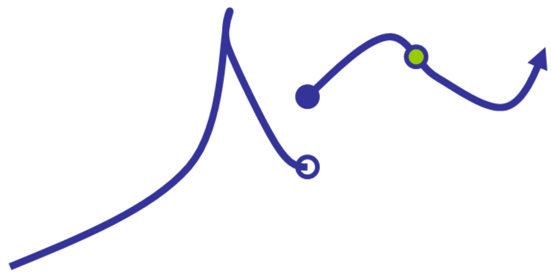
Polynomial function

- Smooth, rounded turns
- Continuous



Not a polynomial function

- Sharp Turn
- Discontinuous



End Behavior

The behavior of the graph of a function to the far left or far right is called the end behavior. Although the graph of a polynomial function may have intervals where it increases or decreases, the graph will eventually continue to positive or negative infinity on both ends, without bound, as it rises or falls.

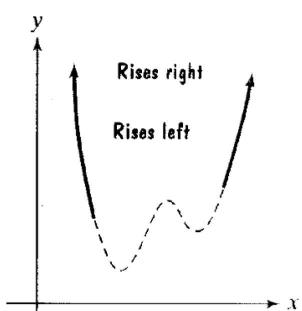
The end behavior of the function is determined by the term with the highest power.

General Guidelines:

When the highest power is EVEN:

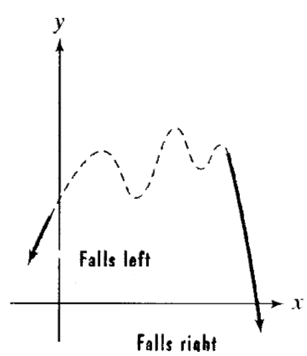
With a positive coefficient:

If the leading coefficient is positive, the graph rises to the left and to the right.



With a negative coefficient:

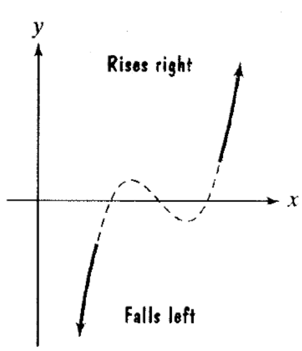
If the leading coefficient is negative, the graph falls to the left and to the right.



When the highest power is ODD:

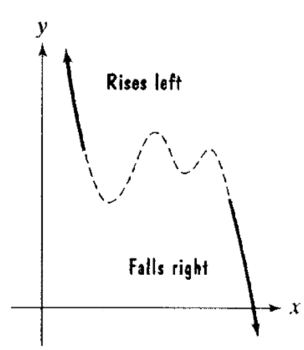
With a positive coefficient:

If the leading coefficient is positive, the graph falls to the left and rises to the right.



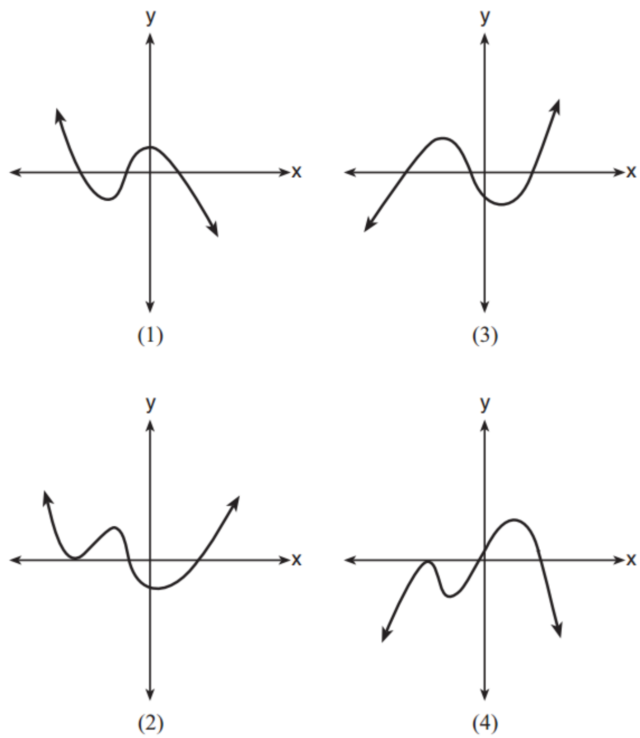
With a negative coefficient:

If the leading coefficient is negative, the graph rises to the left and falls to the right.



Examples:

1. Which graph has the following characteristics?
   - As $x \to \infty,\ f(x) \to \infty$
   - As $x \to -\infty,\ f(x) \to -\infty$



2. Use the leading coefficient test to determine the end behavior of the graph of $g(x) = 3x^2 - x + x^3 - 3$

3. Use the leading coefficient test to determine the end behavior of the graph of $f(x) = -4x^4 - 4x^2$

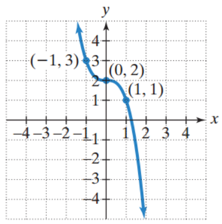
4. Use the leading coefficient test to determine the end behavior of the graph of $f(x) = (x-3)^2$

5. Use the leading coefficient test to determine the end behavior of the graph of $h(x) = 3 - x$

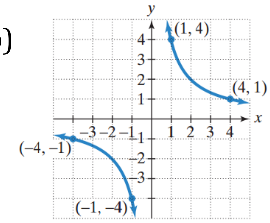
Topic 1 Homework: Symmetry & End Behavior

1. Determine if the following functions are odd, even, or neither. Explain your reasoning.

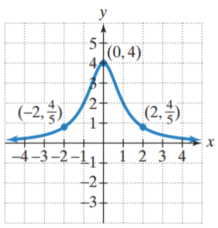
(a)



(b)



(c)



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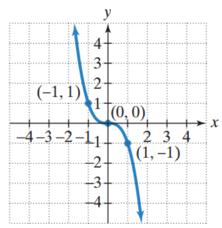


2. Algebraically determine if the following functions are odd, even, or neither.

(a) $f(x) = 2x^3 + x$

(b) $g(x) = 6x^2 + 5x$

(c) $h(x) = x^2 - 3x^4$

(d) $f(x) = \frac{2x^3}{x^2-7}$

(e) $g(x) = \sqrt{2x^2 + 4}$

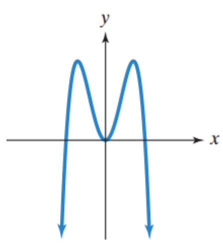
(f) $h(x) = \frac{3x^4}{x^2+5}$

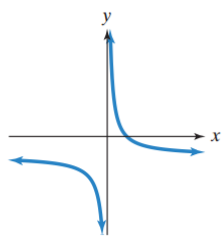
6. Use the leading coefficient test to determine the end behavior of the graph of $g(x) = 4x^3 - 4x + 3x^2$

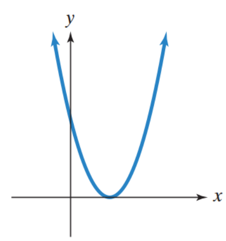
7. Use the leading coefficient test to determine the end behavior of the graph of $g(x) = 2x - 7x^2 + 3$

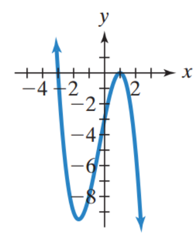
8. Use the leading coefficient test to determine the end behavior of the graph of $g(x) = (x + 6) - (2x - 7)$

9. Which of the following does not represent the graph of a function. Explain your reasoning.









10. Given the partially filled out table below for $f(x)$, fill out the rest of it based on the function type.

(a) Even

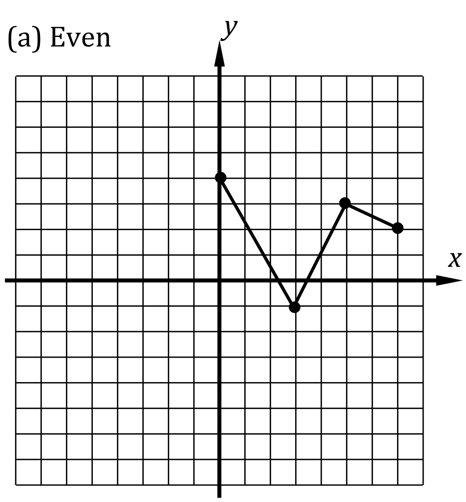
x	−3	−2	−1	0	1	2	3
y	5		−7	4		−4	

(b) Odd

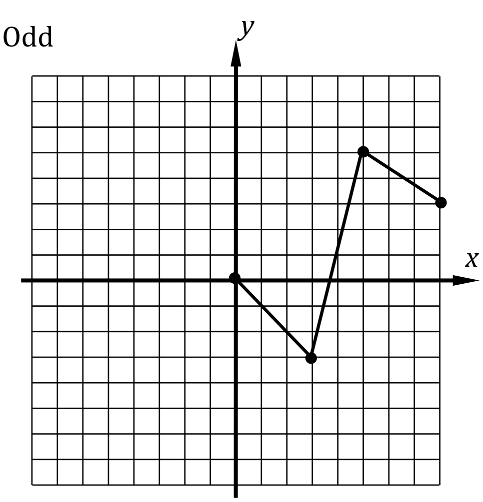
x	−3	−2	−1	0	1	2	3
y	5		−7	0		−4	

11. Half of the graph of $f(x)$ is shown below. Sketch the other half based on the function type.

(a) Even



(b) Odd



12. If $f(x)$ is an even function and $f(3)=5$ then what is the value of $4f(3)+2f(-3)$?

(1) 30 (3) 10

(2) 60 (4) 6

13. Which of the following functions is even?.

(1) $y=x^2-4x$ (3) $y=9-x^2$

(2) $y=|x-6|$ (4) $y=4^x$

14. Even functions have symmetry across the y-axis. Odd function have symmetry across the origin. Can a function have symmetry across the x-axis? Why or why not?