

LESSON 4

LINEAR MODELING

In Common Core Algebra I, you used linear functions to model any process that had a constant rate at which one variable changes with respect to the other, or a constant slope. In this lesson we will review many of the facets of this type of modeling.

Exercise #1: Dia was driving away from NYC at a constant speed of 58 miles per hour. He started 45 miles away.

- (a) Write a linear function that gives Dia's distance, D , from NYC as a function of the number of hours, h , he has been driving.
- (b) If Dia's destination is 270 miles away from NYC, algebraically determine to the nearest tenth of an hour how long it will take Dia to reach his destination.

In *Exercise #1*, it is clear from the context what both the slope and the y -intercept of this linear model are. Although this is often the case when constructing a linear model, sometimes the slope and a point are known, in which case, the point slope form of the a line is more appropriate.

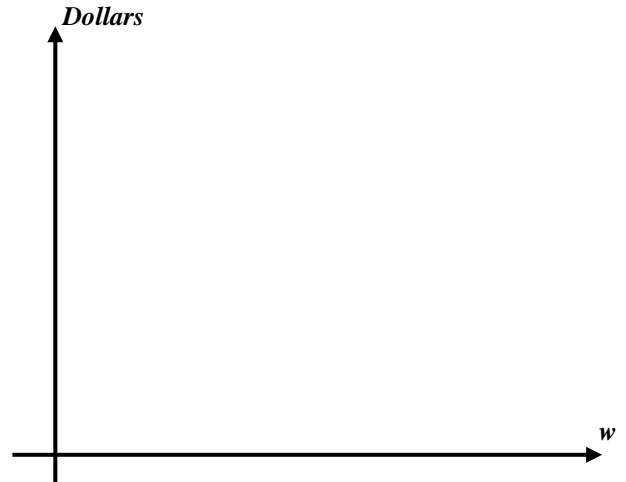
Exercise #2: Eve is trying to model her cell-phone plan. She knows that it has a fixed cost, per month, along with a \$0.15 charge per call she makes. In her last month's bill, she was charged \$12.80 for making 52 calls.

- (a) Create a linear model, in point-slope form, for the amount Eve must pay, P , per month given the number of phone calls she makes, c .
- (b) How much is Eve's fixed cost? In other words, how much would she have to pay for making zero phone calls?

Many times linear models have been constructed and we are asked only to work with these models. Models in the real world can be messy and it is often convenient to use our graphing calculators to plot and investigate their behavior.

Exercise #3: A factory produces widgets. The cost, C , in dollars to produce w widgets is given by the equation $C = 0.18w + 20.64$. Each widget sells for 26 cents. Thus, the revenue gained, R , from selling these widgets is given by $R = 0.26w$.

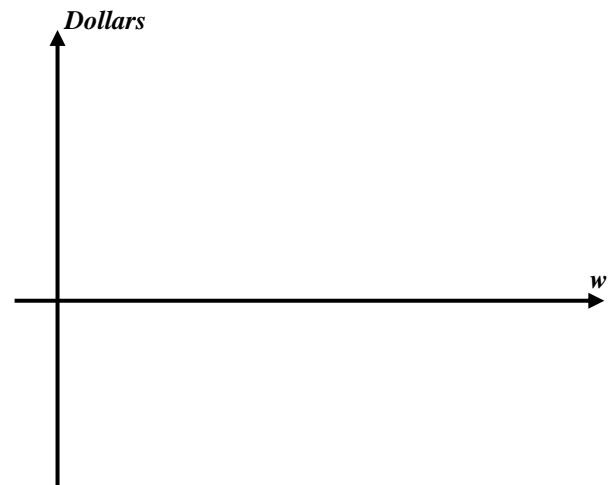
- (a) Use your graphing calculator to sketch and label Each of these linear functions for the interval $0 \leq w \leq 500$. Label the y-axis with its scale.



- (b) Use your calculator's **INTERSECT** command to determine the number of widgets, w , that must be produced for the revenue to equal the cost.

- (c) If profit is defined as the revenue minus the cost, create an equation in terms of w for the profit, P .

- (d) Using your graphing calculator, sketch a graph of the profit over the interval $0 \leq w \leq 1000$. Use a **TABLE** on your calculator to determine an appropriate **WINDOW** for viewing. Label the x and y intercepts of this line on the graph



- (e) What is the minimum number of widgets that must be sold in order for the profit to reach at least \$40. Illustrate this on your graph.

LESSON 4 HOMEWORK

___1. Which of the following would model the distance, D , a driver is from Chicago if they are heading *towards* the city at 58 miles per hour and started 256 miles away?

(1) $D = 256t + 58$

(3) $D = 58t + 256$

(2) $D = 256 - 58t$

(4) $D = 58 - 256t$

___2. The cost, C , of producing x -bikes is given by $C = 22x + 132$. The revenue gained from selling x -bikes is given by $R = 350x$. If the profit, P , is defined as $P = R - C$, then which of the following is an equation for P in terms of x ?

(1) $P = 328x - 132$

(3) $P = 328x + 132$

(2) $P = 372x + 132$

(4) $P = 372x - 132$

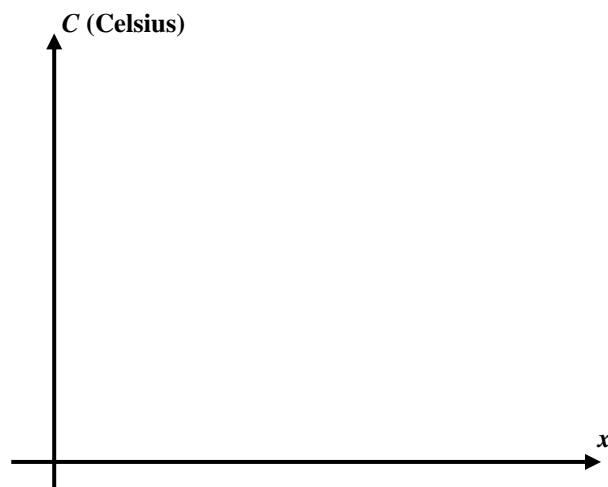
3. The average temperature of the planet is expected to rise at an average rate of 0.04 degrees Celsius per year due to global warming. The average temperature in the year 2000 was 14.71 degrees Celsius. The average Celsius temperature, C , is given by $C = 14.71 + 0.04x$, where x represents the number of years since 2000.

(a) What will be the average temperature in the year 2100?

(b) Algebraically determine the number of years, x , it will take for the temperature, C , to reach 20 degrees Celsius. Round to the nearest year.

(c) Sketch a graph of the average yearly temperature below for the interval $0 \leq x \leq 2000$. Be sure to label your y-axis scale as well as two points on the line. (the y-intercept and on additional point).

(d) What does this model project to be the average global temperature in 2200?



4. Fabio is driving west away from Albany and towards Buffalo along Interstate 90 at a constant rate of speed of 62 miles per hour. After driving for 1.5 hours, Fabio is 221 miles from Albany.

(a) Write a linear model for the distance, D , that Fabio is away from Albany as a function of the number of hours, h , that he has been driving. Write your model in point-slope form, $D - D_1 = m(h - h_1)$.

(b) Rewrite this model in slope-intercept form, $D = mh + b$.

(c) How far was Fabio from Albany when he started his trip?

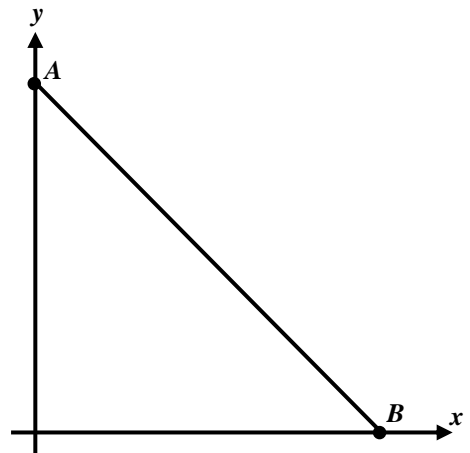
(d) If the total distance from Albany to Buffalo is 290 miles, determine how long it takes for Fabio to reach Buffalo. Round your answer to the nearest tenth of an hour.

5. A particular rocket taking off from the Earth's surface uses fuel at a constant rate of 12.5 gallons per minute. The rocket initially contains 225 gallons of fuel.

(a) Determine a linear model, in $y = ax + b$ form, for the amount of fuel, y , as a function of the number minutes, x , that the rocket has burned.

(b) Below is a general sketch of what the graph of your model should look like. Using your calculator, determine the x and y intercepts of this model and label them on the graph at points A and B respectively.

(c) The rocket must still contain 50 gallons of fuel when it hits the stratosphere. What is the maximum number of minutes the rocket can take to hit the stratosphere? Show this point on your graph by also graphing the horizontal line $y = 50$ and showing the intersection point.



Answers to Lesson 4 - Homework

1) (2)

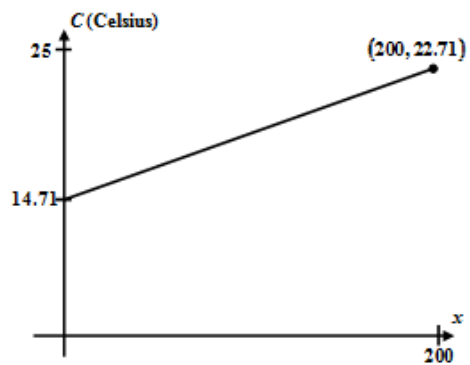
2) (1)

3) (a) 18.71°C

(b) approximately 132 years

(d) 22.71°C

(c)



4) (a) $D - 221 = 62(h - 1.5)$

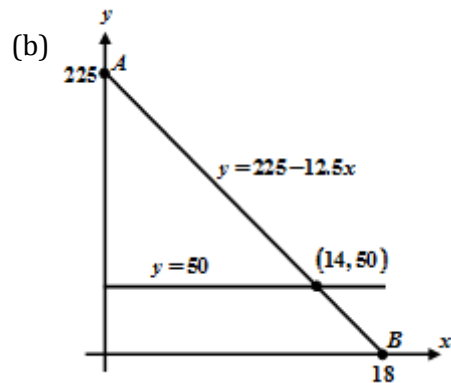
(b) $D = 62h + 128$

(c) 128 miles; which is the y-intercept of this model

(d) approximately 2.6 hours

5) (a) $y = -12.5x + 225$

(c) As shown on the graph, the maximum number of minutes is 14, after that the rocket will contain less than 50 gallons of fuel.



LESSON 5

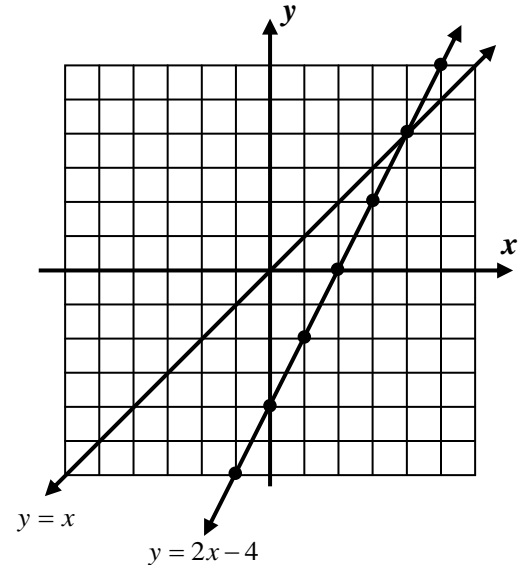
INVERSE OF LINEAR FUNCTIONS

Recall that functions have inverses that are also functions if they are one-to-one. With the exception of horizontal lines, all linear functions are one-to-one and thus have inverses that are also functions. In this lesson we will investigate these inverses and how to find their equations.

Exercise #1: On the grid below the linear function $y = 2x - 4$ is graphed along with the line $y = x$.

(a) How can you quickly tell that $y = 2x - 4$ is a one-to-one function?

(b) Graph the inverse of $y = 2x - 4$ on the same grid. Recall that this is easily done by switching the x and y coordinates of the original line.



(c) What can be said about the graphs of $y = 2x - 4$ and its inverse with respect to the line $y = x$?

(d) Find the equation of the inverse in $y = mx + b$ form.

(e) Find the equation of the inverse in $y = \frac{x+b}{a}$ form.

As we can see from part (e) in *Exercise #1*, inverses of linear functions include the inverse operations of the original function but in reverse order. This gives rise to a simple method of finding the equation of any inverse. **Simply switch the x and y variables in the original equation and solve for y .**

___ **Exercise #2:** Which of the following represents the equation of the inverse of $y = 5x - 20$.

(1) $y = -\frac{1}{5}x + 20$

(3) $y = \frac{1}{5}x - 4$

(2) $y = \frac{1}{5}x - 20$

(4) $y = \frac{1}{5}x + 4$

Although this is a simple enough procedure, certain problems can lead to common errors when solving for y . Care should be taken with each algebraic step.

___ **Exercise #3:** Which of the following represents the inverse of the linear function $y = \frac{2}{3}x + 8$?

(1) $y = \frac{3}{2}x - 8$

(3) $y = -\frac{3}{2}x + 8$

(2) $y = \frac{3}{2}x - 12$

(4) $y = -\frac{3}{2}x + 12$

___ **Exercise #4:** What is the y -intercept of the inverse of $y = \frac{3}{5}x - 9$?

(1) $y = 15$

(3) $y = 9$

(2) $y = \frac{1}{9}$

(4) $y = -\frac{5}{3}$

Sometimes we are asked to work with linear functions in their point-slope form. The method of finding the inverse and plotting it, though, do not change just because the linear equation is written in a different form.

___ **Exercise #5:** Which of the following would be an equation for the inverse of $y + 6 = 4(x - 2)$?

(1) $y - 2 = \frac{1}{4}(x + 6)$ (3) $y - 6 = -4(x + 2)$

(2) $y - 2 = -\frac{1}{4}(x + 6)$ (4) $y + 2 = -4(x - 6)$

___ **Exercise #6:** Which of the following points lies on the graph of the inverse of $y - 8 = 5(x + 2)$? Explain your choice.

(1) $(8, -2)$ (3) $(-10, 40)$

(2) $(-8, 2)$ (4) $(-2, 8)$

___ **Exercise #7:** Which of the following linear functions would *not* have an inverse that is also a function? Explain how you made your choice.

(1) $y = x$ (3) $y = 2$

(2) $2y = x$ (4) $y = 5x - 1$

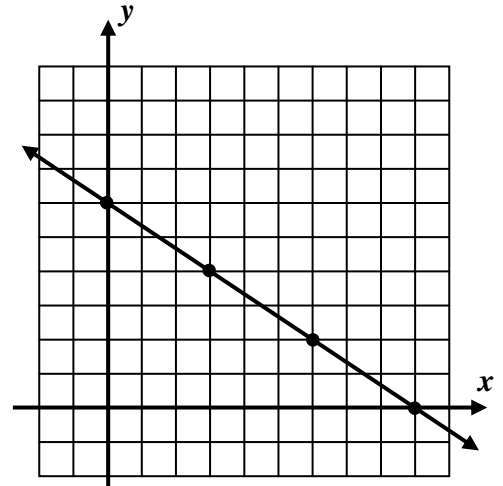
7. A linear function is graphed below. Answer the following questions based on this graph.

(a) Write the equation of this linear function in $y = mx + b$ form.

(b) Sketch a graph of the inverse of this function on the same grid.

(c) Write the equation of the inverse in $y = mx + b$ form.

(d) What is the intersection point of this line with its inverse?



8. A car traveling at a constant speed of 58 miles per hour has a distance of y -miles from Poughkeepsie, NY, given by the equation $y = 58x + 24$, where x represents the time in hours that the car has been traveling.

(a) Find the equation of the inverse of this linear function in $y = \frac{(x-a)}{b}$ form. (b) Evaluate the function you found in part (a) for an input of $x = 227$.

(c) Give a physical interpretation of the answer you found in part (b). Consider what the input and output of the inverse represent in order to answer this question.

9. Given the general linear function $y = mx + b$, find an equation for its inverse in terms of m and b .

Answers to Lesson 5 - Homework

1) (3)

2) (1)

3) (2)

4) (4)

5) (3)

6) (4)

7) (a) $y = -\frac{2}{3}x + 6$

(b) Graph a line by switching the x, and y-coordinates from the given line.

(c) $y = -\frac{3}{2}x + 9$

(d) $\left(\frac{18}{5}, \frac{18}{5}\right)$

8) (a) $y = \frac{x-24}{58}$ (b) $y = 3.5$

(c) In the original function x represented the time and y represented the distance. For the inverse, these two must be switched, thus x represents the distance and y represents the time. So, the interpretation of (b) is that it takes 3.5 hours to reach a distance of 227 miles from Poughkeepsie.

9) $y = \frac{1}{m}x - \frac{b}{m}$ or $y = \frac{x-b}{m}$

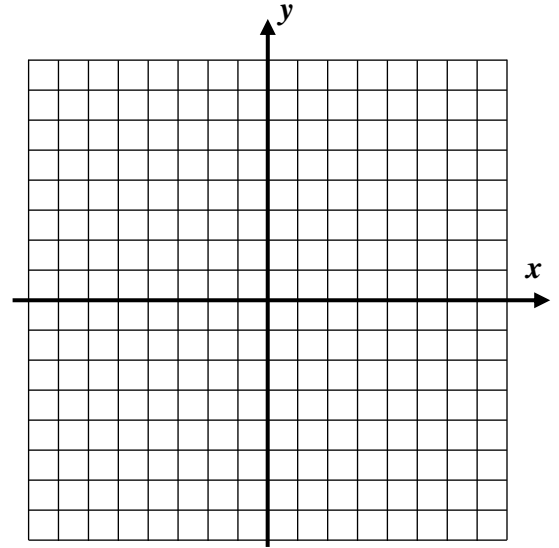
LESSON 6 PIECEWISE LINEAR FUNCTIONS

Functions expressed algebraically can sometimes be more complicated and involve different equations for different portions of their domains. These are known as piecewise functions (they come in pieces). If all of the pieces are linear, then they are known as piecewise linear functions.

Exercise #1: Consider the piecewise linear function given by the formula $f(x) = \begin{cases} x - 3 & -3 \leq x < 0 \\ \frac{1}{2}x + 4 & 0 \leq x \leq 4 \end{cases}$

(a) Create a table of values below and graph the function.

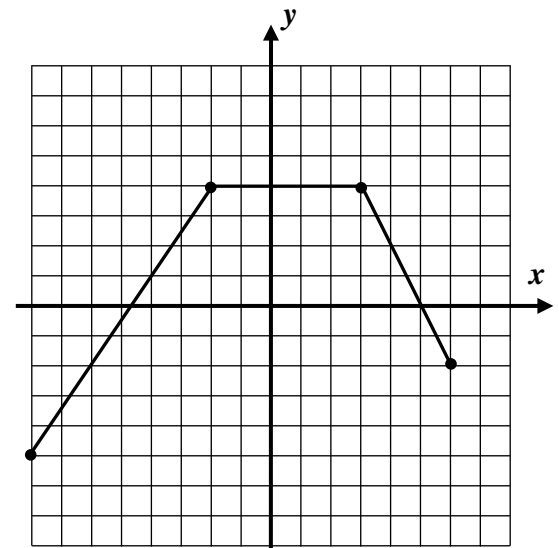
x	-3	-2	-1	0	1	2	3	4
$f(x)$								



(b) State the range of f using interval notation.

Not only should we be able to graph piecewise functions when we are given their equations, but we should also be able to translate the graphs of these functions into equations.

Exercise #2: The function $f(x)$ is shown graphed below. Write a piecewise linear formula for the function. Be sure to specify both the formulas and the domain intervals over which they apply.

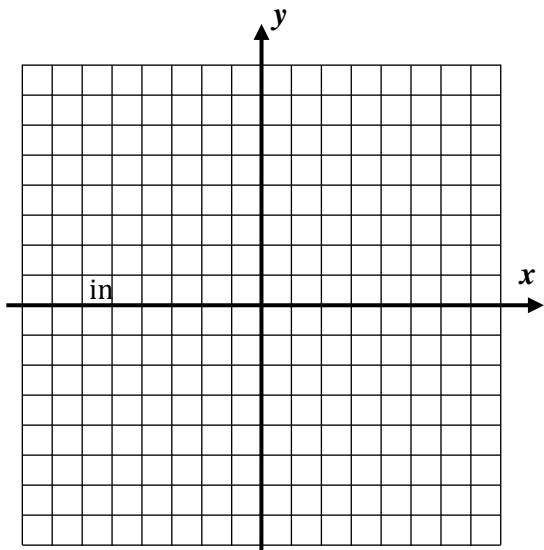


Piecewise equations can be challenging algebraically. Sometimes information that we find from them can be misleading or incorrect.

Exercise #3: Consider the piecewise linear function $g(x) = \begin{cases} 5 - x & x < 2 \\ \frac{1}{2}x + 2 & x \geq 2 \end{cases}$

- (a) Determine the y -intercept of this function algebraically. Why can a function have only one y -intercept? (b) Find the x -intercepts of each individual linear equation.

(c) Graph the piecewise linear function below.



(d) Why does your graph contradict the answers you found part (b)?

(e) How can you resolve the fact that the algebra seems to contradict your graphical evidence of x -intercepts?

Exercise #4: For the piecewise linear function $f(x) = \begin{cases} -2x + 10 & x \leq 0 \\ 5x - 1 & x > 0 \end{cases}$, find all solutions to the equation $f(x) = 1$ algebraically.

LESSON 6 HOMEWORK

1. For $f(x) = \begin{cases} 5x - 3 & x < -2 \\ x + 8 & -2 \leq x < 3 \\ \frac{1}{3}x + 7 & x \geq 3 \end{cases}$ answer the following questions.

(a) Evaluate each of the following by carefully applying the correct formula:

(i) $f(2)$

(ii) $f(-4)$

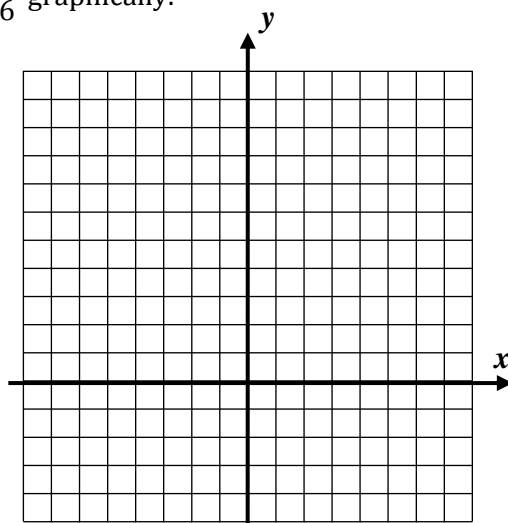
(iii) $f(3)$

(iv) $f(0)$

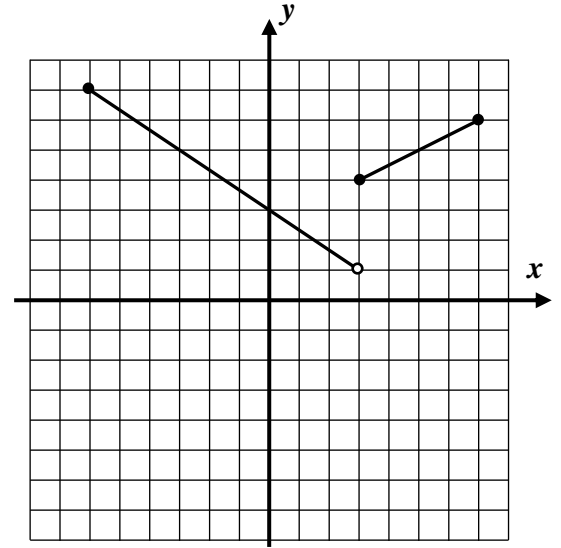
(b) The three linear equations have y -intercepts of -3 , 8 , and 7 respectively. Yet, a function can have only one y -intercept. Which of these is the y -intercept of this function? Explain how you made your choice.

(c) Calculate the average rate of change of f over the interval $-3 \leq x \leq 9$. Show the calculations that lead to your answer.

2. Determine the range of the function $g(x) = \begin{cases} x + 4 & -2 \leq x < 2 \\ -\frac{3}{2}x + 9 & 2 < x \leq 6 \end{cases}$ graphically.

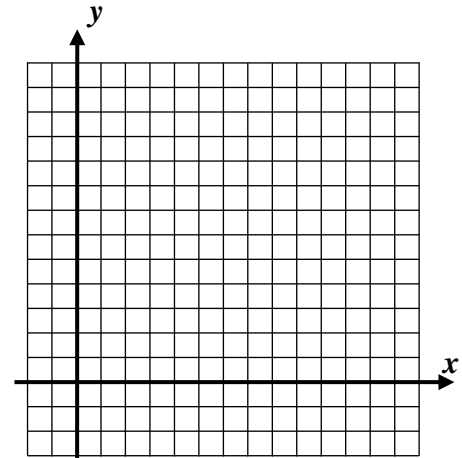


3. Determine a piecewise linear equation for the function $f(x)$ shown below. Be sure to specify not only the equations, but also the domain intervals over which they apply.



4. Step functions are piecewise functions that are constants (horizontal lines) over each part of their domains. Graph the following step function.

$$f(x) \begin{cases} -2 & 0 \leq x < 3 \\ 3 & 3 \leq x < 5 \\ 7 & 5 \leq x < 10 \\ 5 & 10 \leq x \leq 12 \end{cases}$$



5. Find all x -intercepts of the function $g(x) = \begin{cases} 2x + 8 & -5 \leq x < -1 \\ -\frac{1}{2}x - 4 & -1 \leq x < 1 \\ -4x + 10 & 1 \leq x \leq 4 \end{cases}$ algebraically. Justify your work by showing your algebra. Be sure to check your answers versus the domain intervals to make sure each solution is valid.

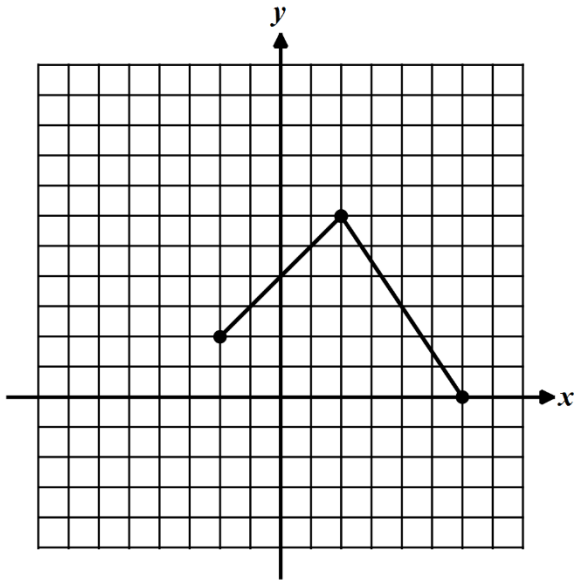
Answers to Lesson 6 - Homework

1) (a) (i) 10 (ii) -23 (iii) 8 (iv) 8

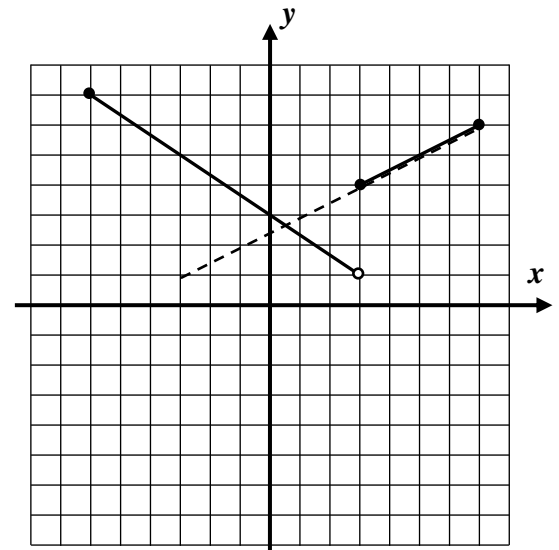
(b) The y -intercept of this function is 8. This is due to the fact that the y -intercept is the output of the function when the input, x , is zero. This was calculated in (iv) above.

(c) $f(-3) = -18; f(9) = 10$
 $AROC = \frac{7}{3}$

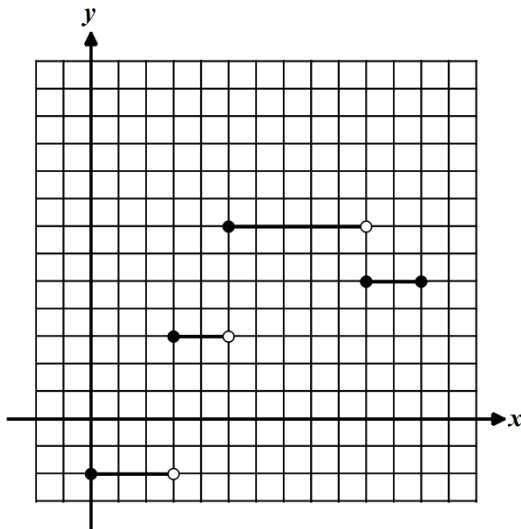
2) Range = $\{0 \leq y \leq 6\}$



$$3) f(x) = \begin{cases} -\frac{2}{3}x + 3 & -6 \leq x < 3 \\ \frac{1}{2}x + \frac{5}{2} & 3 \leq x \leq 7 \end{cases}$$



4)



5) $2x + 8 = 0; y - \text{int}: x = -4$

- **This is a valid solution because it falls in the interval $-5 \leq x < -1$.**

$-\frac{1}{2}x - 4 = 0; y - \text{int}: x = -8$

- **This is a not a valid solution because it does not fall in the interval $-1 \leq x < 1$.**

$-4x + 10 = 0; y - \text{int}: x = \frac{5}{2}$

- **This is a valid solution because it falls in the interval $1 \leq x \leq 4$.**

LESSON 7

SYSTEMS OF LINEAR FUNCTIONS

Systems of equations, or more than one equation, arise frequently in mathematics. To solve a system means to find all sets of values that simultaneously make all equations true. You have solved systems of linear equations in the last two Common Core math courses, but we will add to their complexity in this lesson.

Exercise #1: Solve the following system of equations by: (a) substitution and (b) by elimination.

(a)
$$\begin{aligned} 3x + 2y &= -9 \\ 2x + y &= -7 \end{aligned}$$

(b)
$$\begin{aligned} 3x + 2y &= -9 \\ 2x + y &= -7 \end{aligned}$$

You should be very familiar with solving two-by-two systems of linear equations (two equations and two unknowns). In this lesson, we will extend the method of **elimination** to linear systems of three equations and three unknowns. These linear systems serve as the basis for a field of math known as **Linear Algebra**.

Exercise #2: Consider the three-by-three system of linear equations shown below. Each equation is numbered in this first exercise to help keep track of our manipulations.

$$\begin{array}{l} (1) \quad 2x + y + z = 15 \\ (2) \quad 6x - 3y - z = 35 \\ (3) \quad -4x + 4y - z = -14 \end{array}$$

(a) The **addition property of equality** allows us to add two equations together to produce a third valid equation. Create a system by adding equations (1) and (2) and (2) and (3).

(b) Use this new two-by-two system to solve the three-by-three.

In three-by-three systems we may need to use the **multiplication property of equality** before we can eliminate variables.

Exercise #3: Solve the following system of equations. Show all steps.

$$4x + y - 3z = -6$$

$$-2x + 4y + 2z = 38$$

$$5x - y - 7z = -19$$

Exercise #4: Solve the system of equations. Show all steps.

$$4x - 2y + 3z = 23$$

$$x + 5y - 3z = -37$$

$$-2x + y + 4z = 27$$

LESSON 7
HOMEWORK

1. The sum of two numbers is 5 and the larger difference of the two numbers is 39. Find the two numbers by setting up a system of two equations with two unknowns and solving algebraically.

2. Algebraically, find the intersection points of the two lines whose equations are shown below.

$$4x + 3y = -13$$

$$y = 6x - 8$$

3. Show that $x = 10$, $y = 4$, and $z = 7$ is a solution to the system below *without* solving the system formally.

$$x + 2y + z = 25$$

$$4x + 2y + z = 25$$

$$-2x - y + 8z = 32$$

4. In the following system, the value of the constant c is unknown, but it is known that $x = 8$ and $y = 4$ are the x and y values that solve this system. Determine the value of c . Show how you arrived at your answer.

$$-5x + 2y + 3z = 81$$

$$x - y + z = -1$$

$$2x - y + cz = 35$$

5. Solve the following system of equations. Carefully show how you arrived at your answers.

$$4x + 2y - z = 21$$

$$-x - 2y + 2z = 13$$

$$3x - 2y + 5z = 70$$

6. Algebraically solve the following system of equations. There are two variables that can be readily eliminated, but your answers will be the same no matter which you eliminate first.

$$2x + 5y - z = -35$$

$$x - 3y + 4z = 31$$

$$-3x + 2y + 2z = -23$$

7. Algebraically solve the following system of equations. This system will take more manipulation because there are no variables with coefficients equal to 1.

$$2x + 3y - 2z = 33$$

$$4x + 5y + 3z = 54$$

$$-6x - 2y - 8z = -50$$

Answers to Lesson 7 - Homework

1) The two numbers are -17 and 22 .

2) The point of intersection is $\left(\frac{1}{2}, -5\right)$.

3) Prove by substituting the given values into all three equations and prove equality.

4) $z = 11; c = 5$

5) $x = 9, y = -4, z = 7$

6) $x = 3, y = -8, z = 1$

7) $x = 8, y = 5, z = -1$