2.2 Angles Formed by Tangents, Secants, and Chords

There are other angles that fall inside and outside the circle, formed by various rays and line segments that intersect the circle.

A tangent is a line or ray that intersects the circle at exactly one point. In the diagram at the right, \( PA \) is a tangent to the circle.

A secant is a line or ray that passes through the circle, intersecting the circle at two points. In the figure, \( PBC \) is a secant of the circle.

The different angles formed by such lines and rays are detailed below.

<table>
<thead>
<tr>
<th>Type of Angle</th>
<th>Identifying Elements</th>
<th>Diagram</th>
<th>Formula</th>
<th>Example</th>
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</thead>
<tbody>
<tr>
<td>Angle formed by tangent and chord</td>
<td>The vertex is on the circle at the point of tangency; one side is a chord; one is a tangent.</td>
<td></td>
<td>Measure of the angle is one-half the intercepted arc.</td>
<td>If ( m\overarc{BC} = 110 ), ( m\angle PBC = \frac{1}{2}(110) ). ( m\angle PBC = 55 )</td>
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<tr>
<td>Angle formed by a secant and chord</td>
<td>The vertex is on the circle but one side of the angle lies inside the circle and one side lies outside. Angle formulas cannot be used.</td>
<td></td>
<td>Measure of the angle equals 180 – the measure of the adjacent inscribed angle. ( \angle CTP ) is a straight angle, therefore, ( m\angle PTH = 180 - m\angle CTH ).</td>
<td>If ( m\angle CH = 116 ), ( m\angle CTH = \frac{1}{2} CH ) ( = \frac{1}{2}(116) ) ( = 58 ). ( m\angle PTH = 180 - 58 ) ( = 122 )</td>
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<tr>
<td>Angle formed by two chords intersecting within a circle</td>
<td>Its vertex is inside the circle, its sides are parts of chords. The chords form vertical angles.</td>
<td></td>
<td>Measure of an angle formed by two intersecting chords equals one-half the sum of the measures of the intercepted arcs of the angle and its vertical angle. ( m\angle MEJ = \frac{1}{2}(m\overarc{JM} + m\overarc{KN}) ).</td>
<td>If ( m\overarc{MJ} = 56 ) and ( m\overarc{KN} = 164 ), ( m\angle MEJ = \frac{56 + 164}{2} ) ( = \frac{1}{2}(220) ) ( = 110 )</td>
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| Angle formed by a tangent and a radius | The vertex is on the circle at the point of tangency.                                | ![Diagram](image) | Angle formed is always a right angle.  
\[ \angle OAB = 90 \] |         |
| Angle formed by two tangents intersecting outside the circle | The vertex is outside the circle and the sides of the angle are tangents.  
The tangents divide the circle’s 360° into two arcs. | ![Diagram](image) | Measure of the angle formed by two tangents equals one-half the difference of the measure of the intercepted arcs.  
\[ \angle QPR = \frac{1}{2} (\angle QTR - \angle QR) \]  
OR  
The angle formed by two tangents outside the circle is supplementary to the measure of the minor arc of the circle.  
\[ \angle QPR = 180 - \angle QR \] | If \( \angle QR = 114 \), then  
\[ \angle QTR = 360 - 114 = 246 \]  
\[ \angle QPR = \frac{246 - 114}{2} = \frac{132}{2} = 66 \] |
| Angle formed by two secants intersecting outside the circle | The vertex is outside the circle and the sides of the angle are secants. | ![Diagram](image) | The measure of an angle formed by two secants equals one-half the difference of the measures of the intercepted arcs.  
\[ \angle RPS = \frac{1}{2} (\angle SR - \angle SV) \] | If \( \angle SR = 128 \) and  
\( \angle SV = 32 \), then  
\[ \angle RPS = \frac{128 - 32}{2} = \frac{96}{2} = 48 \] |
| Angle formed by a secant and a tangent intersecting outside the circle | The vertex is outside the circle.  
One ray of the angle is a secant and one is a tangent. | ![Diagram](image) | The measure of the angle formed by a secant and a tangent equals one-half the difference of the measures of the intercepted arcs.  
\[ \angle APC = \frac{1}{2} (\angle CA - \angle AB) \] | If \( \angle CA = 140 \) and  
\( \angle AB = 52 \), then  
\[ \angle APC = \frac{140 - 52}{2} = \frac{88}{2} = 44 \] |