Chapter 5

Radicals

Chapter 5: Radicals

Exponent Laws Review

EXPONENT LAWS

1. $x^a \cdot x^b =$ ______
2. $\frac{x^a}{x^b} =$ ______
3. $(x^a)^b =$ ______
4. $x^{\frac{m}{n}} =$ ______ (For integers *m* and *n*)
5. $x^{-a} =$ ____ and $\frac{1}{x^{-a}} =$ ____
6. $(x \cdot y)^a =$ ______
7. $x^0 =$ ______

Problems using ***exponent laws*** can be challenging, but it is important to apply the laws in a systematic manner. Be careful to not make any mistakes.

Exercise #1: Simplify each of the following expressions. Leave no negative exponents in your answers.

a) $\frac{x^3 \cdot x^4}{(x^5)^2}$

b) $\frac{(x^2y)^4}{x^5y^7}$

c) $\frac{x^{-2}y^4}{x^{-6}y}$

d) $\frac{(x^{-3}y^{-4})^2}{(xy^3)^{-4}}$

In the last exercise, all of the powers were integers. In the next exercise, we introduce fractional powers. Remember, though, that they will still follow the exponent rules above. If needed, use your calculator to help add and subtract the powers.

Exercise #2: Simplify each of the following expressions. Write each without the use of negative exponents.

a) $\frac{x^{\frac{1}{3}} \cdot x^{\frac{1}{2}}}{x^{\frac{1}{6}}}$

b) $\frac{(x^{\frac{1}{2}})^5}{x^{\frac{3}{2}} \cdot x^3}$

c) $\frac{(4x^{\frac{2}{3}})^3}{32x^8}$

We must not forget that fractional exponents have an equivalent interpretation as roots. We should be able to move from one representation to another. A fractional exponent can be represented as ____________________________.

Exercise #3: Rewrite each expression below in both its simplest form and using radical expressions.

a) $x^{\frac{5}{3}}$

b) $\frac{x^{\frac{5}{2}}}{x^{\frac{4}{3}}}$

c) $\frac{1}{x^{\frac{-3}{2}}}$.

d) $\frac{x^3}{\sqrt{x}}$

e) $(8x^5)^{\frac{2}{3}}$

f) $\frac{(27x)^{\frac{1}{3}}}{6\sqrt{x}}$

Exercise #4: Which of the following is equivalent to $\sqrt[3]{8x^7}$?

(1) $8x^{\frac{7}{3}}$ (2) $2x^{\frac{7}{3}}$ (3) $2x^{\frac{3}{7}}$ (4) $8x^{\frac{3}{7}}$

Exercise #5: The expression $\frac{1}{\sqrt{4x}}$ is the same as:

(1) $\frac{1}{2}x^{\frac{-1}{2}}$ (2) $2x^{-1/2}$ (3) $4x^{\frac{1}{2}}$ (4) $\frac{1}{2}x^{1/2}$

Chapter 5: Radicals
Lesson 1: Homework
EXPONENT PRACTICE

FLUENCY

1. Rewrite each of the following expressions in simplest form and without negative exponents.

   a) $\frac{x^3x^7}{(x^2)^3}$ b) $\frac{(x^3y^4)^2}{(x^3y)^3}$ c) $\frac{(2x^3)^5}{8x^{-3}}$

2. Which of the following represents the value of $\frac{a^{-4}}{b^{-2}}$ when $a = 3$ and $b = 2$?

   (1) $\frac{4}{9}$ (2) $\frac{4}{81}$ (3) $\frac{1}{36}$ (4) $\frac{1}{3}$

3. Simplify each expression below so that it contains no negative exponents. Do not write the expressions using radicals.

   a) $\frac{x^{\frac{7}{2}}y^{\frac{1}{2}}}{x^{\frac{3}{4}}y^2}$ b) $\frac{(x^{\frac{1}{3}})^4}{x^{\frac{-2}{3}}}$ c) $(5x^{\frac{2}{3}}y^{\frac{-1}{2}})(2x^2y^{-3})$

4. Which of the following represents the expression $\frac{24x^{\frac{-1}{2}}}{6x^{\frac{5}{2}}}$ written in simplest form?

(1) $\frac{4}{x^3}$ (2) $4x^3$ (3) $\frac{x^2}{4}$ (4) $4x^2$

5. Rewrite each of the following expressions using radicals. Express your answers in simplest form.

a) $(4x)^{\frac{3}{2}}$

b) $x^{\frac{-2}{3}}$

c) $\frac{\sqrt[3]{x}}{\sqrt{x}}$

d) $\frac{\sqrt{x} \cdot x^2}{x^{\frac{5}{3}}}$

6. Which of the following is equivalent to $\frac{5\sqrt{x}}{20x^3}$?

(1) $\frac{1}{4\sqrt{x^3}}$ (2) $\frac{4}{\sqrt{x^5}}$ (3) $\frac{1}{4\sqrt[5]{x^2}}$ (4) $\frac{1}{4\sqrt{x^5}}$

7. When written in terms of a fraction exponent the expression $\frac{\sqrt{x}\cdot x}{x^{-2}}$ is:

(1) $x^{7/2}$ (2) $x^{5/2}$ (3) $x^{-1/2}$ (4) $x^{-3/2}$

8. Expressed as a radical expression, the fraction $\frac{x^{\frac{1}{3}}x^{\frac{1}{2}}}{x^{-1}}$ is

(1) $\frac{1}{\sqrt[6]{x}}$ (2) $\frac{1}{\sqrt[11]{x^6}}$ (3) $\sqrt[11]{x^6}$ (4) $\sqrt[6]{x^{11}}$

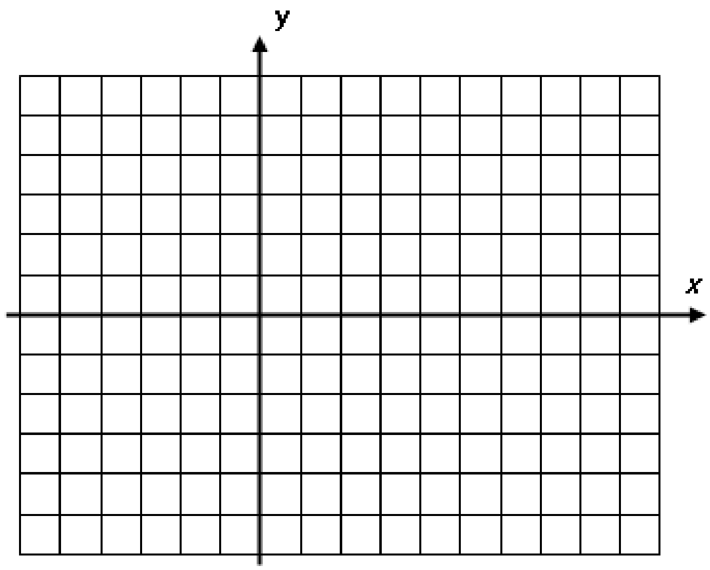
Chapter 5
LESSON 2: SQUARE ROOT FUNCTIONS
COMMON CORE ALGEBRA II

Recall:

Domain: Set of all _________ values. **Range:** Set of all _________ values.

Exercise #1: Consider the two functions $f(x) = \sqrt{x}$ and $g(x) = \sqrt{x+3} - 2$.

a) Graph $f(x)$ on the grid shown. Label its equation.



b) Using your calculator to generate a table of values, graph $y = g(x)$ on the same grid and label its equation. Start your table at $x = -10$ to see certain x-values not in the domain of this function.

c) State the domain and range of each function below using set-builder notation.

$f(x) = \sqrt{x}$ $\qquad$ $g(x) = \sqrt{x+3} - 2$

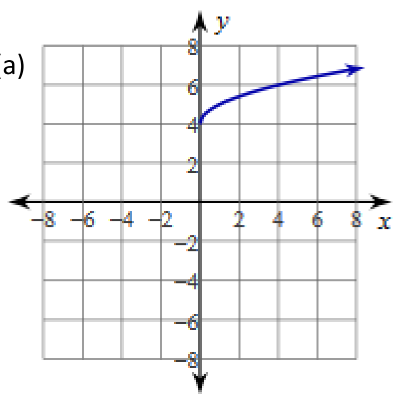
Domain: $\qquad$ ***Domain***:

Range: $\qquad$ ***Range***:

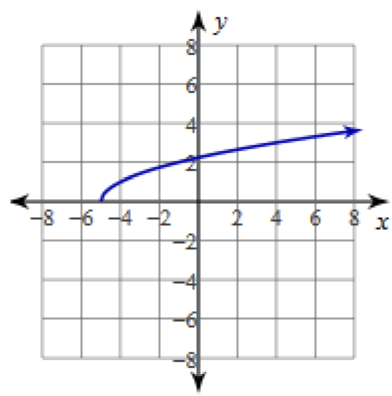
d) Describe the shift from f(x) to g(x).

Exercise #2: Write the equation of the following radical graphs.

(a)

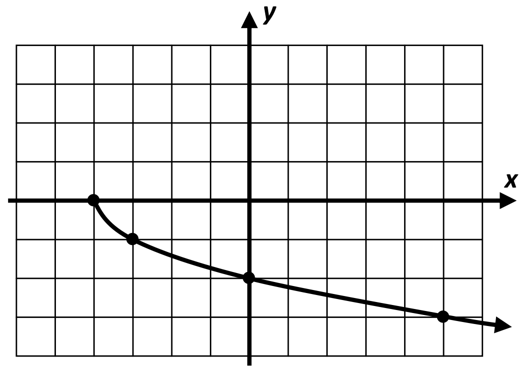


(b)



Exercise #3: Which of the following equations would represent the graph shown below?

1) $y = -\sqrt{x+4}$
2) $y = 4 - \sqrt{x}$
3) $y = \sqrt{x-4}$
4) $y = -\sqrt{x-4}$



As we saw in the first exercise, the domains of square root functions are restricted due to the fact that the square roots of the negative numbers ________________ in the Real Number System.

To find the domain of a radical, we must set the radicand (number inside the radical) to be greater than or equal to zero.

Exercise #4: Which of the following values of *x* does ***not*** lie in the domain of the function $y = \sqrt{x-5}$? Explain why it does not lie there.

1) $x = 6$
2) $x = 2$
3) $x = 5$
4) $x = 7$

Explanation:

____

____

____

____

____

____

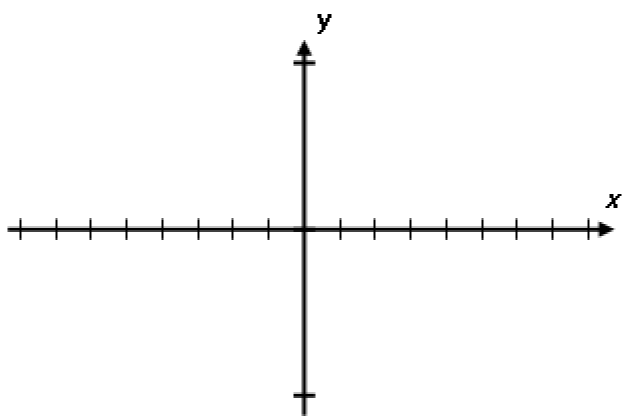
____

Exercise #5: Determine the domain for each of the following square root functions. Show an inequality that justifies your work.

a) $y = \sqrt{x+2}$ b) $y = \sqrt{3x-2}$ c) $y = \sqrt{8-2x}$

Exercise #6: Consider the function $f(x) = \sqrt{x^2+4x-12}$.

1. Use your calculator to sketch the functions on the axes given.



2. Set up and solve a quadratic inequality that yields the domain of $f(x)$.

Chapter 5
Lesson 2 Homework
Square Root Functions

FLUENCY

1. Which of the following represents the domain and range of $y = \sqrt{x - 5} + 7$?

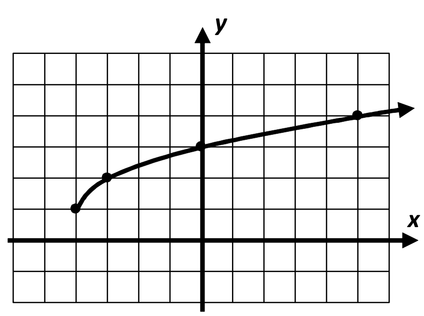
    a. Domain: [-5, ∞), Range: [7, ∞)
    b. Domain: [5, ∞), Range: [7, ∞)
    c. Domain: (-7, ∞), Range: (5, ∞)
    d. Domain: {7, ∞), Range {5, ∞)

2. Which of the following values of *x* is not in the domain of $y = \sqrt{1 - 3x}$?

    a. $x = \frac{1}{3}$
    b. $x = -1$
    c. $x = 0$
    d. $x = 4$

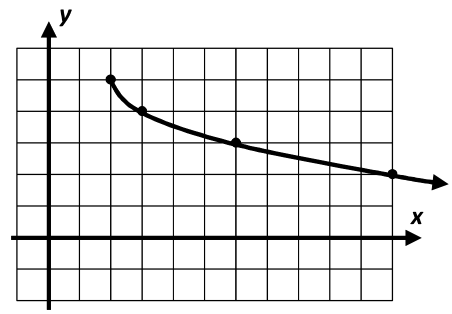
3. Which of the following equations describes the graph shown below?

    a. $y = \sqrt{x + 4} + 1$
    b. $y = \sqrt{x - 4} - 1$
    c. $y = \sqrt{x + 4} - 1$
    d. $y = \sqrt{x - 4} + 1$



4. Which equation below represents the graph shown?

a. $y = \sqrt{x-2} - 5$
b. $y = -\sqrt{x+2} + 5$
c. $y = -\sqrt{x-2} + 5$
d. $y = \sqrt{x+2} + 5$



5. Determine the domains of each of the following functions. State your answers in set-builder notations.

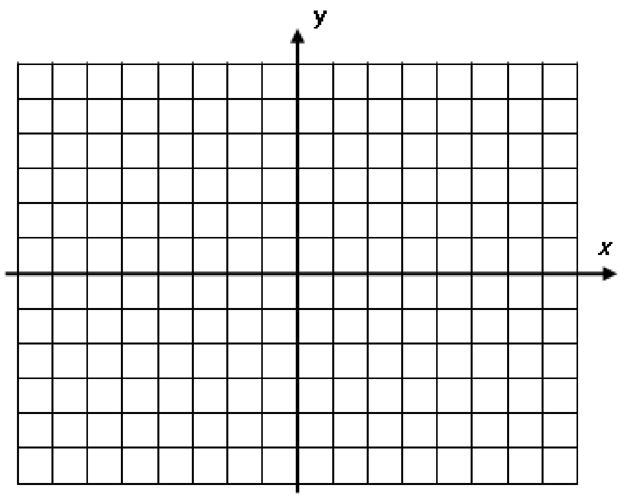
a) $y = \sqrt{x+10}$ b) $y = \sqrt{3x-5}$ c) $y = \sqrt{7-2x}$

6. Set up and algebraically solve a quadratic inequality that results in the domain of each of the following.

a) $y = \sqrt{x^2 - 4x - 5}$ b) $y = \sqrt{9 - x^2}$

7. Consider the function $g(x) = -\sqrt{x+5} + 3$.

   a. Graph the function $y = g(x)$ on the grid shown.



   b. Describe the transformations that have occurred to the graph of $y = \sqrt{x}$ to produce the graph of $y = \sqrt{x}$ to produce the graph of $y = g(x)$. Specify both the transformations and their order.

Chapter 5
LESSON 3: SOLVING SQUARE ROOT EQUATIONS
COMMON CORE ALGEBRA II

Solving Square Root Equations

Solve:

$\sqrt{3x+1}-1=x-4$

Steps:

1. ____________ the radical on one side of the equation.

2. ____________ both sides of the equation to "undo" the square root.

3. ____________ the remaining equation for x.

4. Check!!! Remember you cannot take the square root of a ____________ number.

__Exercise #1__: Solve each of the following square roots equations, which are arranged from less to more complex. Check each equation.

a) $\sqrt{x}=7$

b) $\sqrt{x-3}=5$

c) $\sqrt{2x-1}=4$

d) $3\sqrt{x}-4=20$

e) $2\sqrt{x+5}+7=13$

f) $5\sqrt{3x-2}-4=36$

<u>Exercise #2</u>: Which of the following is the solution to $3\sqrt{\frac{x}{2}}=15$?

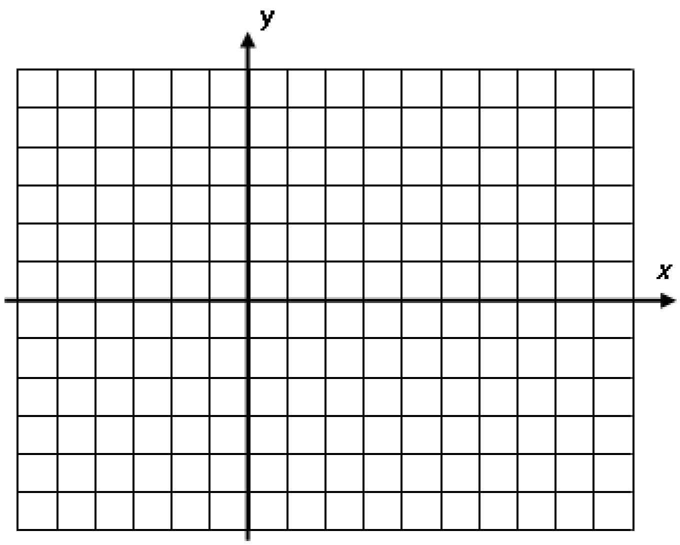
a) $x=12.5$
b) $x=25$
c) $x=50$
d) $x=4050$

A more complicated scenario arises when a square root expression is equal to a linear expression. The next exercise will illustrate both the graphical and algebraic issues involved.

<u>Exercise #3</u>: Consider the system of equations show below.

$y = \sqrt{x+3}$ and $y = x + 1$

a) Solve this system graphically using the grid to the below.



b) Solve this system *algebraically* for only the x-values using substitution below. Remember to CHECK your solution.

Exercise #4: Which function below has a greater x-intercept, f(x) or g(x)? Justify your answer.

$f(x) = 2x^2 - 15x + 25$ $\qquad\qquad$ $g(x) = \sqrt{x+2} - 1$

Exercise #5: Find the solution set of each of the following equations. Be sure to check your work and reject any extraneous roots.

a) $\sqrt{2x} - 3 = x - 3$ $\qquad\qquad$ b) $2x = \sqrt{x+6} - 2$

Chapter 5
Lesson 3 Homework
SOLVING SQUARE ROOT EQUATIONS

FLUENCY

1. Solve each of the following square root equations. Check your answers.

a) $\sqrt{x+2} = 10$

b) $\sqrt{\frac{2x}{3}} = 6$

c) $2\sqrt{x} = 1$

d) $\sqrt{3x+4} = 8$

e) $\frac{1}{2}\sqrt{x} = 24$

f) $5\sqrt{1-5x} - 3 = 27$

g) $\sqrt{x^2 - 10x + 25} = 25$

h) $\sqrt{x+5} = \sqrt{x^2 - 15}$

i) $\sqrt{2x-2} + 1 = x$

Chapter 5 Radicals

Lesson 4

Simplifying Radicals

Simplifying Non-Perfect Square Radicals: When a radical is not a perfect square, it is important to get it into *simplest radical form*.

Rule:

$$\sqrt{Non - Perfect\ Square} = \sqrt{largest\ perfect\ square} \cdot \sqrt{remaining\ factor}$$

In order for a variable to be a perfect square, it must have an even exponent

Let's Try Some…

1.) $5\sqrt{32}$ 2.) $\sqrt{200x^7}$ 3.) $\sqrt{500}$ 4.) $\sqrt{16a^3}$

5.) $\sqrt{3600y^5}$ 6.) $\sqrt{48x^8}$ 7.) $2\sqrt{125}$ 8.) $7\sqrt{52}$

9.) $3\sqrt{45x^2}$ 10.) $9\sqrt{16x^4}$ 11.) $\sqrt{x^5y^8z^{17}}$ 12.) $-9x^5y\sqrt{x^{15}z^{25}}$

13.) If $a > 0$, then $\sqrt{9a^2 + 16a^2}$ equals

(1) $\sqrt{7a}$ (2) $5\sqrt{a}$ (3) 5a (4) 7a

Chapter 5 Radicals

Lesson 4 Homework

Complete the questions below.

In 1 – 6, simplify the given expressions:

1.) $\sqrt{288}$

2.) $3\sqrt{72}$

3.) $3\sqrt{112x^2}$

4.) $4\sqrt{450a^7}$

5.) $\sqrt{200x^{20}}$

6.) $8x\sqrt{675x^{15}y^3}$

7.) In simplest form, $\sqrt{300}$ is equivalent to

(1) $3\sqrt{10}$ (2) $5\sqrt{12}$ (3) $10\sqrt{3}$ (4) $12\sqrt{5}$

Chapter 5: Radicals
Lesson 5
Simplifying Cube Roots

Simplifying Cubed Roots:

$\sqrt[3]{108}$

1. Find the largest perfect cube which will divide evenly into the number under your radical sign. (If you need assistance finding perfect cubes, type y = x^3 into the graphing part of the calculator. The table lists perfect cubes.)

2. Write the number appearing under your radical as the product of the perfect cube and your answer from dividing. Give each number in the product its own radical sign.

3. Reduce the "perfect" radical which you have now created.

Examples: Simplify each cube root.

1.) $\sqrt[3]{-192}$

2.) $\sqrt[3]{750x^8}$

3.) $5\sqrt[3]{864x^3y^4}$

4.) $xy\sqrt[3]{1000x^9y^{10}}$

5.) The expression $\sqrt[3]{64a^{16}}$ is equivalent to

(1) $8a^4$

(2) $8a^8$

(3) $4a^5\sqrt[3]{a}$

(4) $4a\sqrt[3]{a^5}$

6.) Which function has the greater average rate of change over the interval [1,64]?

$f(x) = \sqrt{x}$

$g(x) = \sqrt[3]{x}$

Chapter 5: Radicals
Lesson 5: Homework
Simplifying Cube Roots

Simplify each of the following cube roots.

1.) $\sqrt[3]{54x^3}$

2.) $\sqrt[3]{-48}$

3.) $\sqrt[3]{56y^{13}}$

4.) $6\sqrt[3]{375}$

5.) $-12\sqrt[3]{-432}$

6.) $2\sqrt[3]{1024x^5}$

7.) What is the y-intercept of the function $g(x) = \sqrt[3]{x-8} + 2$?

8.) Is the domain of a cube root function restricted? Justify your answer.