Chapter 6
Complex Numbers

Lesson 1: Imaginary Numbers

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Chapter 6: Complex Numbers
Lesson 1
Imaginary Numbers

Imaginary Numbers

Recall that in the Real Number System, it is not possible to take the square root of a __________________ quantity because whenever a real number is squared it is _________________.

Exercise 1: (a) Algebraically, find the x-intercepts of the quadratic equation $y = x^2 + 1$.

(b) Sketch the graph of the function $y = x^2 + 1$. How does your answer to part (a) relate to the graph?

Since we cannot solve this equation using Real Numbers, we introduce a new number, called *i*, the basis of __________________ ***numbers***. Its definition allows us to now have a result when finding the square root of a _____________ real number. Its definition is given below.

The Definition of the Imaginary Number *i*:

***i* = ______________**

Whenever we have a negative underneath a radical, we must "________________" the negative and make the expression _________________.

Exercise 2: Simplify each of the following square roots in terms of *i*.

(a) $\sqrt{-9}$

(b) $\sqrt{-100}$

(c) $\sqrt{-32}$

(d) $\sqrt{-18}$

(e) $2\sqrt{-250}$

(f) $-5\sqrt{-75}$

(g) $2\sqrt{-243}$

(h) $2\sqrt{-144}$

Powers of i display a remarkable pattern that allow us to simplify large powers of i into one of 4 cases. This pattern is discovered in Exercise 5.

Exercise 3: Simplify each of the following powers of i.

$i^1 = i$ $\quad$ $i^2 =$ $\quad$ $i^3 =$ $\quad$ $i^4 =$

$i^5 =$ $\quad$ $i^6 =$ $\quad$ $i^7 =$ $\quad$ $i^8 =$

We see, then, from this pattern that every power of i is either -1,1,i, or -i. And the pattern will repeat. This follows a cycle.

Exercise 4: From the pattern of Exercise 3, simplify each of the following powers of i.

(a) $i^{38} =$ $\quad$ (b) $i^{21} =$ $\quad$ (c) $i^{83} =$ $\quad$ (d) $i^{40} =$

Multiplying with ***imaginary numbers*** is just like multiplying with a variable. We ____________ ***the coefficients*** and then _________ ***the powers of i***.

Exercise 5: Evaluate. Final answers must be in simplest form.

(a) $9i^5 \bullet 10i^7$

(b) $-12i^2 \bullet 7i^6$

Exercise 6: Which of the following is equivalent to $5i \bullet 6i$?

(1) $30i$

(2) $11i$

(3) -30

(4) -11

Exercise 7: Which of the following is equivalent to $5i^{16} + 3i^{23} + i^{26}$?

(1) $8 + 2i$

(2) $4 - 3i$

(3) $5 - 4i$

(4) $2 + 7i$

Chapter 6: Complex Numbers
Lesson 1: Homework
Imaginary Numbers

1.) The imaginary number *i* is defined as

(1) -1 (2) $\sqrt{-1}$ (3) $\sqrt{-4}$ (4) $(-1)^2$

2.) Which of the following is equivalent to $\sqrt{-128}$?

(1) $8\sqrt{2}$ (2) 8i (3) $-8\sqrt{2}$ (4) $8i\sqrt{2}$

3.) Mrs. Arena made up a game to help her class learn about imaginary numbers. The winner will be the student whose expression is equivalent to –i. Which expression will win the game?

(1) i^{46} (2) i^{47} (3) i^{48} (4) i^{49}

4.) For any power of *i*, the imaginary unit, where b is a whole number, i^{4b+2} equals

(1) 1 (2) i (3) -1 (4) –i

5.) In simplest form, what is the sum of $\sqrt{-9}$ and $\sqrt{-16}$?

6.) Simplify the following radical expressions. Final answers must be in simplest radical form.

(a) $-3\sqrt{-150}$

(b) $i\sqrt{-120}$

(c) $2\sqrt{-12}$

(d) $10\sqrt{-180}$

7.) Simplify each of the following powers of i.

(a) i^{53}

(b) i^{207}

(d) i^{698}

8.) Which of the following is equivalent to $i^7 + i^8 + i^9 + i^{10}$?

(1) 1

(2) 2 + i

(3) 1 – i

(4) 0

Chapter 6: Complex Numbers
Lesson 2
Complex Numbers

Complex Numbers:

All numbers fall into a very broad category known as complex numbers. Complex numbers can always be thought of as a combination of a real number with an imaginary number and will have the form:

________________________ where *a* and *b* are real numbers

We say that *a* is the _____________ part of the number and *bi* is the ________________ part of the number. These two parts, the real and imaginary, ***cannot be*** _______________. Like real numbers, complex numbers may be added and subtracted. The key to these operations is that real components can combine with real components and imaginary with imaginary.

Complex numbers must always be written in _____________________ form, so the imaginary part must be last.

Exercise 1: Find each of the following sums and differences.

(a) (-2 + 7i) + (6 + 2i)

(b) (8 + 4i) + (12 - i)

(c) (5 + 3i) - (2 - 7i)

(d) (-3 + 5i) - (-8 + 2i)

Exercise 2: Which of the following represents the sum of (6+2i) and (-8-5i)?

(1) 5i   (3) 2 + 3i

(2) -2 - 3i   (4) -5i

Complex numbers are ________________ ***under multiplication.*** We multiply complex numbers similar to the way that we would multiply algebraic expressions, by __________________.

Exercise 3: Find the following products. Write each of your answers as a complex number in the form *a + bi*.

(a) (3 + 5i)(7 + 2i)   (b) (-2 + 6i)(3 - 2i)

(c) (4 + i)(-5 - 3i)   (d) 2(5-2i) + 3i(5 - 6i)

(e) $(5 + 3i)^2$

A ***conjugate*** is a binomial formed by _____________________ the second term of a binomial. For example, the conjugate of x + y is _________________.

Exercise 4: For the examples below, find the product of the complex number and its conjugate.

(a) (5 – i)

(b) (9 + 2i)

(c) (6 + 7i)

(d) (1 + 10i)

Exercise 5: Determine the result of the calculation below in simplest *a + bi* form.

(5 + 2i)(-3 + i) + 4i(2 + 3i)

Exercise 6: Which of the following products would be a purely real number?

(1) (4+2i)(3-i)

(2) (-3+i)(-2+4i)

(3) (5+2i)(5-2i)

(4) (6+3i)(6+3i)

Exercise 7: Given *i* is an imaginary unit, write in simplest form:

$(2+yi)^2$

Exercise 8: Simplify $ai(2i – 8i)^2$, where i is the imaginary unit.

Chapter 6: Complex Numbers
Lesson 2: Homework
Complex Numbers

1.) Stacey and Rachel are playing a game with complex numbers. If Stacey has a score of 5 – 4i and Rachel has a score of 3 + 2i, what is their total score?

(1) 8 + 6i (2) 8 + 2i (3) 8 – 6i (4) 8 – 2i

2.) Expressed in a + bi form, $(1 + 3i)^2$ is equivalent to

(1) 10 + 6i (2) -8 + 6i (3) 10 – 6i (4) -8 – 6i

3.) Find each of the following products in simplest a + bi form.

(a) (5 – 2i)(-1 + 7i) (b) (-4 – i)(-2 + 6i)

4.) Which of the following is equivalent to 3(5 + 2i) – 2(3 – 6i)?

(1) 9 + 18i (2) 21 + 8i (3) 9 – 6i (4) 21 – 2i

5.) Find the product of the complex number and its conjugate.

(a) (5 + 7i)

(b) (-3 + 8i)

6.) Perform the following complex calculation. Express your answer in simplest a + bi form.

(8 + 5i)(3 + 2i) – (4 + i)(4 – i)

7.) If f(x) = x^2, then f(2 – 3i) equals

(1) -5

(2) -5 – 12i

(3) 13 – 12i

(4) 13

8.) What is the product of $5 + b\sqrt{-36}$ and $1 - b\sqrt{-49}$, expressed in simplest a + bi form?

Chapter 6: Complex Numbers
Lesson 3
Quadratic Formula

Quadratic Formula:

One way to solve quadratic equations that are not ________________ is the ***Quadratic Formula***. The quadratic formula can also be used to solve __________ type of quadratic equation, even if it is factorable.

Standard Form of Quadratic Equation: ______________________________

Quadratic Formula: ____________________________________

Example:

1.) Solve: $x^2 - 5x = -15$

Steps:

1) Write equation in standard form.

2) List all the values for a, b, and c.

3) Write out the quadratic formula.

4) Substitute the values for a, b, & c into the quadratic formula.

5) Simplify your answer.

Recall: A complex number ***must*** be written in a+bi form.

Exercise 1: Use the quadratic formula to find all solutions to the following equation. Express your answer in simplest *a + bi* form.

$$x^2 - 4x + 29 = 0$$

As long as our solutions can include complex numbers, then any quadratic equation can be solved for two roots.

Exercise 2: Solve each of the following quadratic equations. Express your answer in simplest *a + bi* form.

(a) $x^2 - 5x + 30 = 7x - 10$

(b) $x^2 + 16x + 15 = 10x + 4$

Exercise 3: Which of the following represents the solutions to the equation $x^2 - 10x + 20 = 0$?

(1) $x = 5 \pm \sqrt{10}$

(2) $x = 5 \pm \sqrt{5}$

(3) $x = -5 \pm \sqrt{10}$

(4) $x = -5 \pm \sqrt{5}$

Exercise #4: Solve each of the following quadratic equations by using the quadratic formula. Place all answers in simplest form.

(a) $3x^2 + 5x + 1 = 0$

(b) $x^2 - 8x = -13$

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Lesson 3: Homework
Quadratic Formula

1.) Solve each of the following quadratic equations. Express all complex solutions in simplest a + bi form.

(a) $x^2 + 4x + 20 = 12x - 5$

(b) $x^2 - 2x - 1 = 0$

(c) $5x^2 + 8x - 2 = 0$

(d) $8x^2 + 36x + 24 = 12x + 5$

2.) The solutions to the equation $x^2 + 6x + 11 = 0$ are

(1) $x = -3 \pm i\sqrt{2}$ (3) $x = -6 \pm i\sqrt{11}$

(2) $x = -3 \pm 2i\sqrt{2}$ (4) $x = -6 \pm 2i\sqrt{11}$

3.) Which of the following quadratics, if graphed, would lie entirely above the x-axis?

(1) $y = 2x^2 + x - 21$ (3) $y = x^2 - 4x + 7$

(2) $y = x^2 - x - 6$ (4) $y = x^2 - 10x + 16$

Challenge:

4.) For what values of c will the quadratic $y = x^2 + 6x + c$ have no real zeroes? Set up and solve an inequality for this problem.

Chapter 6: Complex Numbers
Lesson 4
Discriminant

The Discriminant of a Quadratic:

Since the roots of a quadratic can be found using ________________________, if *a*, *b*, and *c* are all rational numbers, the quantity under the square root, $b^2 - 4ac$, dictates what type of numbers the roots of a quadratic (and its x-intercepts) turn out to be.

Discriminant: _________________________

The discriminant tells us about the "_______________" of the roots of a quadratic equation. It also gives us information about the _______________ of a quadratic.

Discriminant	"Nature" of Roots	Graphs
1.) $b^2 - 4ac < 0$ (negative)		
2.) $b^2 - 4ac = 0$		
3.) $b^2 - 4ac > 0$ **AND** is a perfect square		
4.) $b^2 - 4ac > 0$ **AND** is a non -perfect square		

Exercise 1: By using only the discriminant, determine the number and nature of the roots of each of the following quadratics.

(a) $2x^2 + 7x - 4 = 0$

(b) $x^2 - 8x + 25 = 0$

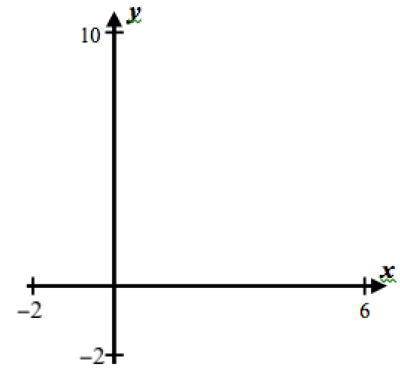
(c) $4x^2 + 4x + 1= 0$

(d) $x^2 + 6x + 15 = 0$

(e) $4x^2 - 4x - 7= 0$

(f) $3x^2 - 7x + 2 = 0$

Exercise 2: Consider the quadratic function whose equation is $y=x^2 - 4x + 4$. Determine the number of x-intercepts of this quadratic from the value of its discriminant. Then, sketch its graph on the axes given.



Exercise 3: Which of the following parabolas has two unequal, rational x-intercepts?

(1) $y = x^2 - 2x - 1$

(2) $y = x^2 + 2x - 15$

(3) $y = x^2 - 8x + 16$

(4) $y = x^2 - 3x + 5$

Exercise 4: For what values of *a* will the parabola $y = ax^2 + 4x + 2$ not cross the x-axis?

Exercise 5: If a quadratic equation with real coefficients has a discriminant of 3, then the two roots must be

(1) real and rational

(2) real and irrational

(3) imaginary

(4) equal

Chapter 6: Complex Numbers
Lesson 4: Homework
Discriminant

1.) For each of the following quadratic equations, determine the value of the discriminant and then state the nature of the roots.

(a) $3x^2 + x - 1 = 0$

(b) $2x^2 - x + 7 = 0$

(c) $x^2 + 2x + 3 = 0$

(d) $2x^2 - 4 = 0$

2.) The equation $2x^2 + 8x + n = 0$ has imaginary roots when n is equal to

(1) 10 (2) 8 (3) 6 (4) 4

3.) If $b^2 - 4ac < 0$, the roots of the equation $ax^2 + bx + c = 0$ must be

(1) real, irrational, and unequal

(2) real, rational, and unequal

(3) real, rational, and equal

(4) imaginary

4.) Which equation has imaginary roots?

(1) $x(5 + x) = 8$

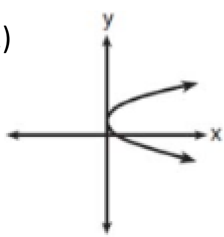
(2) $x(5 - x) = -3$

(3) $x(x + 6) = -10$

(4) $(2x + 1)(x - 3) = 7$

5.) If zero is the value of the discriminant of the equation $ax^2 + bx + c = 0$, which graph best represents $y = ax^2 + bx + c$?

(1)



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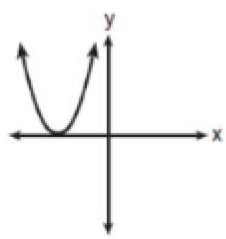

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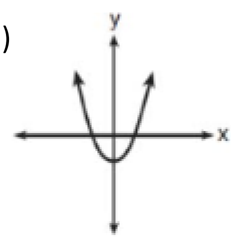

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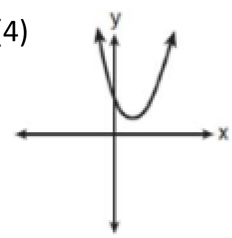

6.) For what values of *a* will the parabola $y = ax^2 + 4x + 2$ not cross the x-axis?

7.) For what value(s) of *b* will the parabola $y = 8x^2 + bx + 2$ have roots that are real, rational, and equal?