Chapter 8
Polynomial Functions and Their Graphs

Chapter 8
Lesson 1: Even & Odd Functions

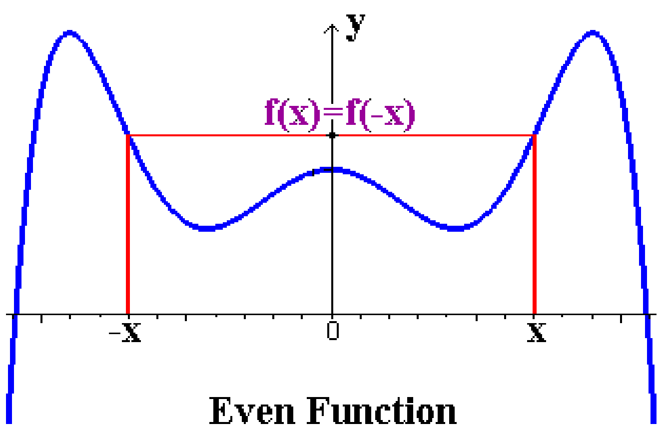
One way to classify functions is based on ***symmetries.***

EVEN AND ODD FUNCTIONS

A function is known as **even** if $f(-x) = f(x)$ for every value of *x* in the domain of $f(x)$.

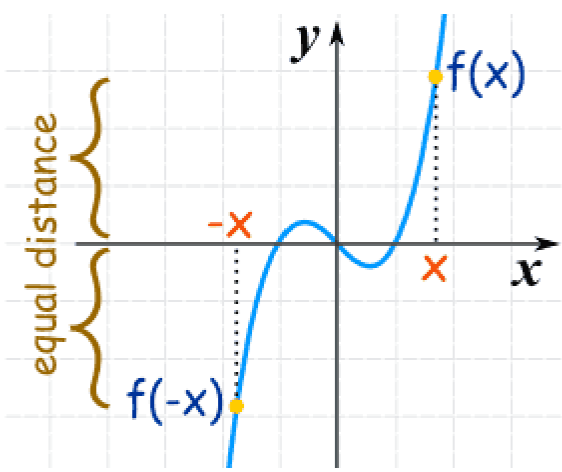
A function is known as **odd** if $f(-x) = -f(x)$ every value of *x* in the domain of $f(x)$.

Even Functions: Even functions are symmetric with the ____________________. In other words, they are a mirror image on either side of the ______________________________.



Even Function

Odd Functions: Odd functions are symmetric with the ____________________.



Testing for Even & Odd Functions:

If a function is **even** its graph is *symmetrical with respect to the* _____________. If a function is **odd** its graph is *symmetric with respect to the* ______________. Just because a function has an even degree, it does not mean that the function is even. The same is true for an odd function.

Testing for symmetry with no graph to reference:

The function y = f(x) is even if f(-x) = ____________.

The function y = f(x) is odd if f(-x) =______________.

Examples: Determine whether the function is even, odd, or neither. If the function is even or odd, indicate the type of symmetry the function has.

1.) $f(x) = x^3 - x$

2.) $f(x) = x^4 + 1$

3.) $f(x) = 2x^3 - x^2 + 1$

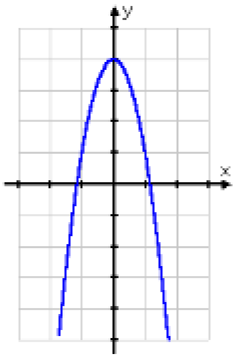
4.) $f(x) = x^6 - x^3 + 1$

5.) $f(x) = -x^2 + 7$

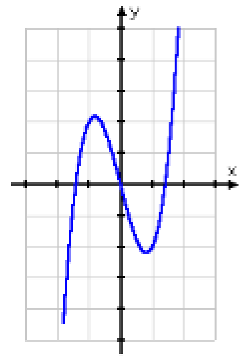
6.) $f(x) = -3x^5 + 8x^3$

Examples: Determine whether the function is even, odd, or neither. Justify your response.

7.)



8.)



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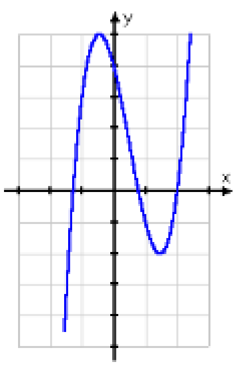
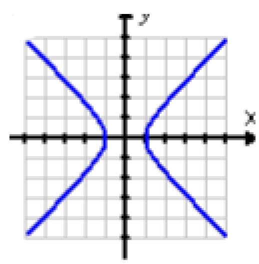


10.)



Examples: Determine whether the function is even, odd, or neither based on the given equation. Justify your response.

11. $f(x) = x^5 - x^3$

12. $f(x) = x^2 - x^4$

13. $f(x) = x^2 - x^4 + 1$

14. $f(x) = x^5 + 2x^4 - 3x^3$

Chapter 8
Lesson 1: Even & Odd Functions
Homework

Examples: Determine whether each function is even, odd, or neither. Then discuss what type of symmetry the function has.

1.) $f(x) = x^3 - 3x$

2.) $g(x) = 5x^4 - 9x^2 + 2$

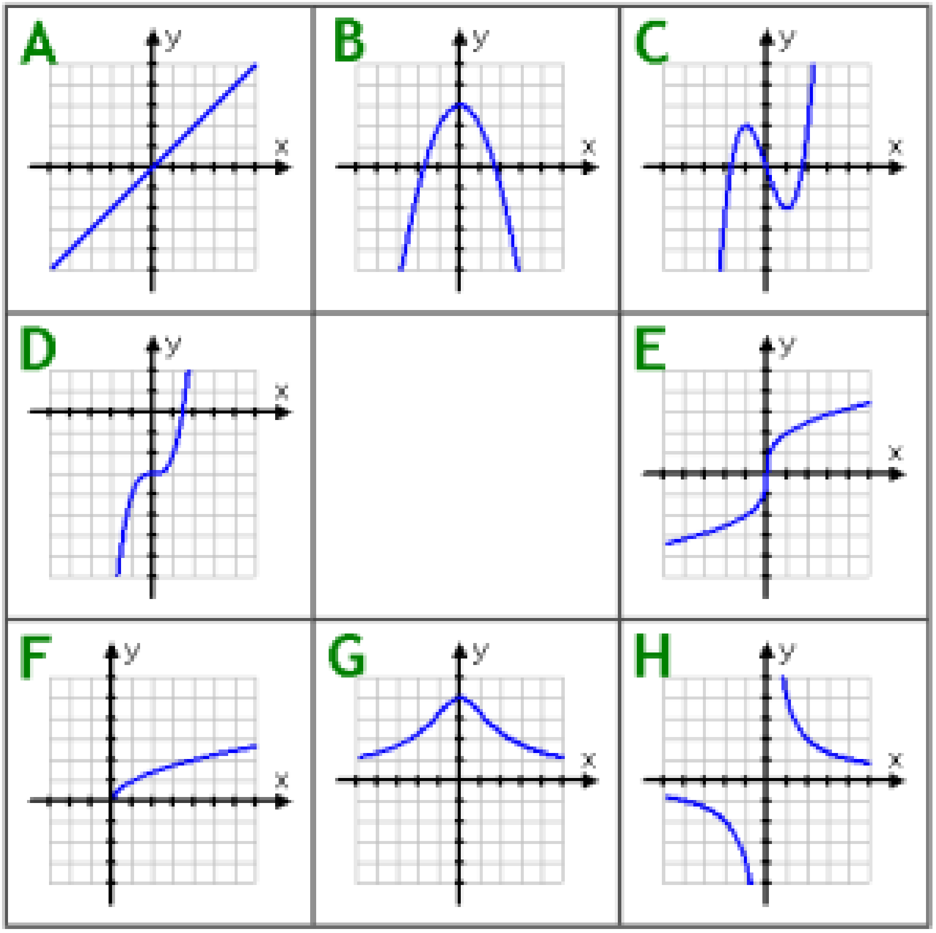
3.) $f(x) = x^3 + x^2 + x + 1$

4.) $f(x) = 2x^3 - 4x$

5.) $h(x) = 7x^2 - 11$

6.) $f(x) = \frac{x}{x^2-1}$

Example 7: Determine whether the function is even, odd, or neither. Justify your response.



Chapter 8
Lesson 2: End Behaviors of Graphs

Do Now:

1.) Use your graphing calculator to sketch the graphs of the following functions:

(a) $g(x) = x^4 - 2$

(b) $f(x) = x^6 - x^2 - 3x + 1$

(c) $h(x) = -x^4 - 5x + 6$

(d) $f(x) = x^3 - x$

(e) $g(x) = -x^5 - 6x^2$

(f) $g(x) = x^3$

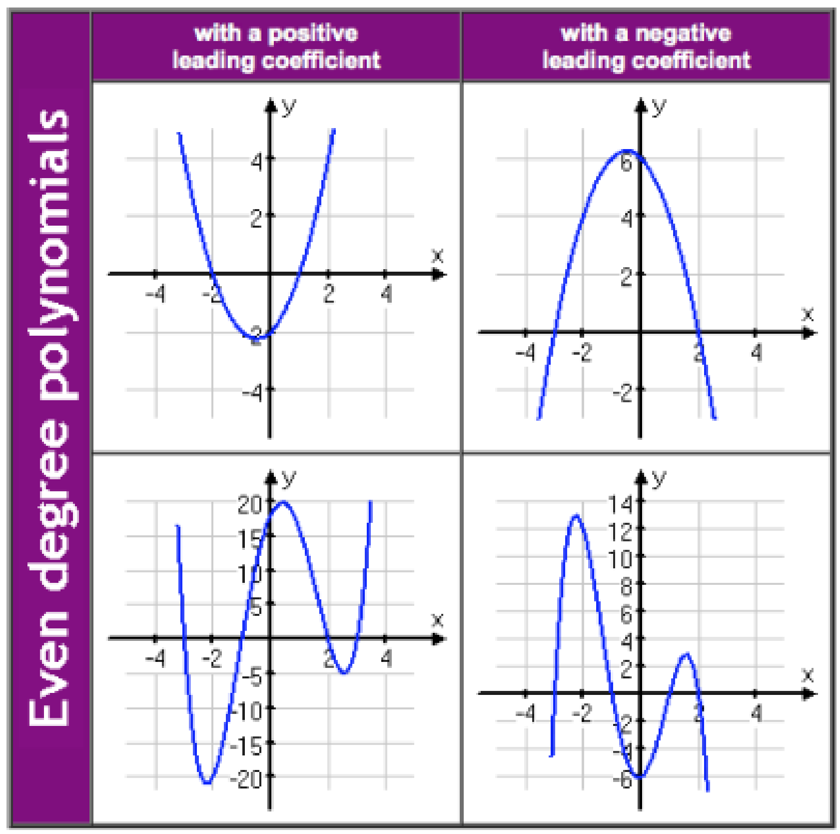
2.) What occurs at the end of each graph in a function whose highest exponent is even (a – c)? What occurs at the end of each graph in a function whose highest exponent is odd (d – f)?

End Behavior:

The *end behavior* of a polynomial function is the behavior of the graph of a function as x approaches positive or negative infinity. Basically, it means what is occurring at each of the ends of the polynomial.

As we can see from the ***Do Now***, *even degree* polynomials are either “up” on both ends if the leading coefficient is ____________________ or “down” on both ends if the leading coefficient is ____________________.

Arrow Notation:



Even/Positive Coefficient:

as x --> -∞,   f(x) --> ______

as x --> ∞,   f(x) --> ______

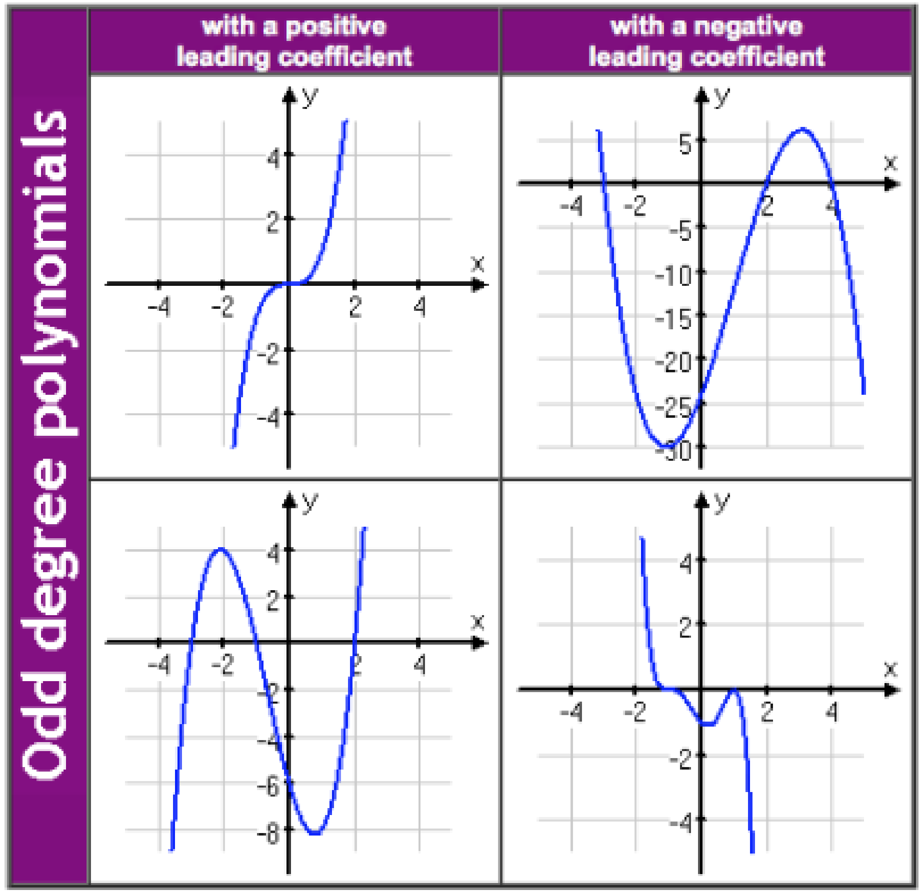
Even/Negative Coefficient:

as x --> -∞,   f(x) --> ______

as x --> ∞,   f(x) --> ______

As we can see from the ***Do Now***, *odd degree* polynomials have ends that go in _______________ directions. If they start “down” and go “up,” they are _____________ polynomials. If they start “up” and go “down,” they are _______________ polynomials.

Arrow Notation:



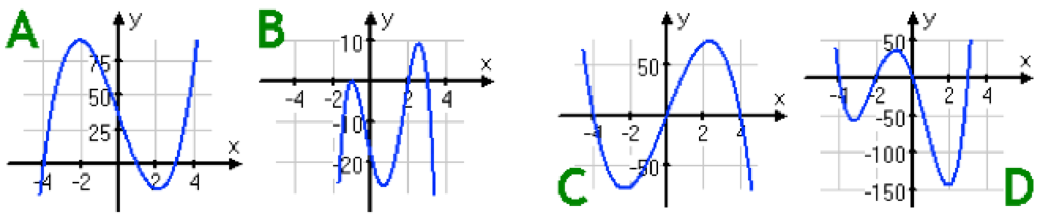
Odd/Positive Coefficient:

as x --> -∞,   f(x) --> _____
as x --> ∞,   f(x) --> _____

Odd/Negative Coefficient:

as x --> -∞,   f(x) --> _____
as x --> ∞,   f(x) -->_____

1.) Which of the following could be the graph of a polynomial function whose leading term is "-3x^4"? Write in words how you selected your answer.



2.) Describe the end behavior of $f(x) = 3x^7 + 5x + 1004$. Justify your answer.

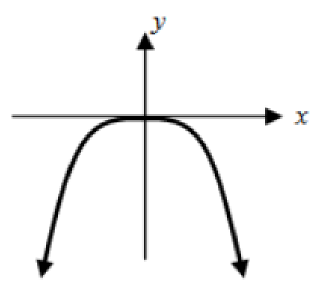
3.) Which of the following functions is shown in the graph? Explain your choice.

(1) y = -4x^7

(3) y = 6x^8

(2) y = -3x^{10}

(4) y = 5x^9



4.) Describe the end behavior of the following functions.

(a) $g(x) = x^3 + 10x^2 + 32x + 34$

(b) $f(x) = -x^2 - 8x - 15$

(c) $f(x) = -x^4 + x^2 + 2$

(d) $y = -x^5 + 4x^3 - 5x - 4$

5.) Which of the following functions decreases as the input values approach both negative infinity and positive infinity?

(1) $f(x) = x^3 - 4x^2 + x$

(3) $g(x) = -2x^3 - 4x^2 + 9$

(2) $h(x) = x^4 - 4x^3 + 2x + 8$

(4) $r(x) = -x^4 + 9x^3 + x^2 + 8x + 2$

Chapter 8
Lesson 2: End Behaviors of Graphs
HOMEWORK

Fluency:

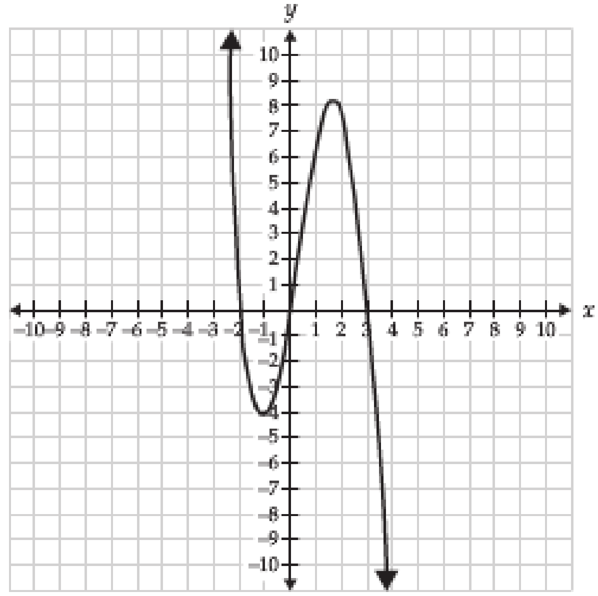
1.) A graph of a polynomial function is shown. Which of the following statements is true of the graphed function?

(1) The degree is odd; the leading coefficient is positive.

(2) The degree is even; the leading coefficient is positive.

(3) The degree is odd; the leading coefficient is negative.

(4) The degree is even; the leading coefficient is negative.



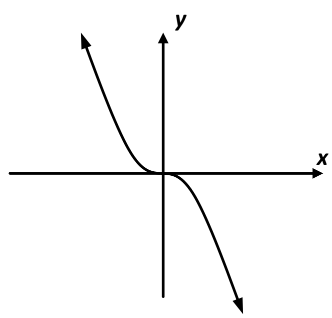
2.) Determine which of the following equations could represent the graph shown below. Explain your choice.

(1) $y = x^2$

(2) $y = x^3$

(3) $y = -x^4$

(4) $y = -x^5$



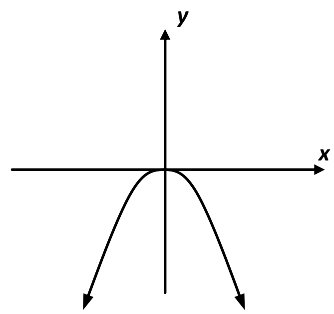
3.) The graph below could be the long-run behavior for which of the following functions?

(1) $y = 2x^2 - 7x + 1$

(2) $y = 4x^3 + 2x^2 - 6x + 4$

(3) $y = -5x^4 + 3x^3 - 2x^2 + x + 9$

(4) $y = -3x^5 - 4x^2 + 2x + 1$



4.) Describe the end behavior of the following functions.

(a) $y = 6x^5 - 4x^3 + x - 120$

(b) $y = 10 - 8x^2$

(c) $y = 5x^4 + 2x^2$

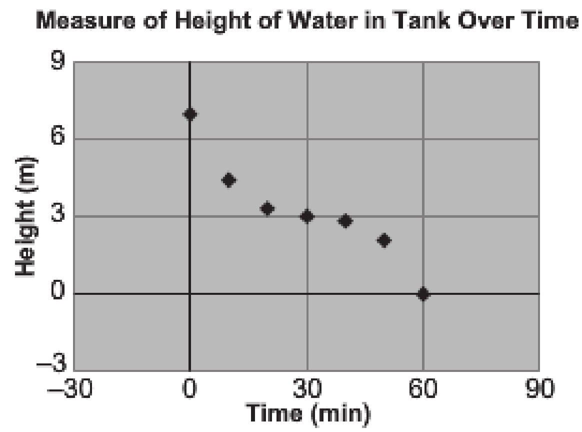
(d) $y = -3x^5 + 2x^4 - 4x + 7$

Reasoning:

5.) Use your knowledge of polynomials and the given table to answer the following questions.

As a water tank is being emptied, the height of the remaining water is measured every ten minutes. The table provides the numbers and the scatter gives a visual sense of the data.

Time Elapsed (minutes)	Height of Water in Tank (meters)
0	7
10	4.42
20	3.27
30	3.09
40	2.85
50	2.11
60	0



(a) This data can be modeled by a polynomial function. Determine if the degree should be even or odd. Justify your response.

(b) Is the leading coefficient of the polynomial that can be used to model this data positive or negative? Explain.

Chapter 8

Lesson 3: Graphing Polynomial Functions by Zeros

Graphs of Functions:

One way that we can graph functions are by using zeros of the function. This is where the function crosses the x-axis. When doing this, it is important to note any high or low points of the graph as well as where the function is increasing or decreasing.

To Graph a Polynomial Function:

1) Find the functions' __________________ (roots)
2) State the _________________ (substitute 0 for x and solve)
3) State the ___________ behavior

In order to graph: plot the roots and y intercepts. Draw in the end behavior. Connect the ends of the graphs by making sure you go through all intercepts. (Some graphs will be tangent to the x-axis at certain points. When this happens, the graph will not go through the x-intercept but rather hit the axis and turn around)

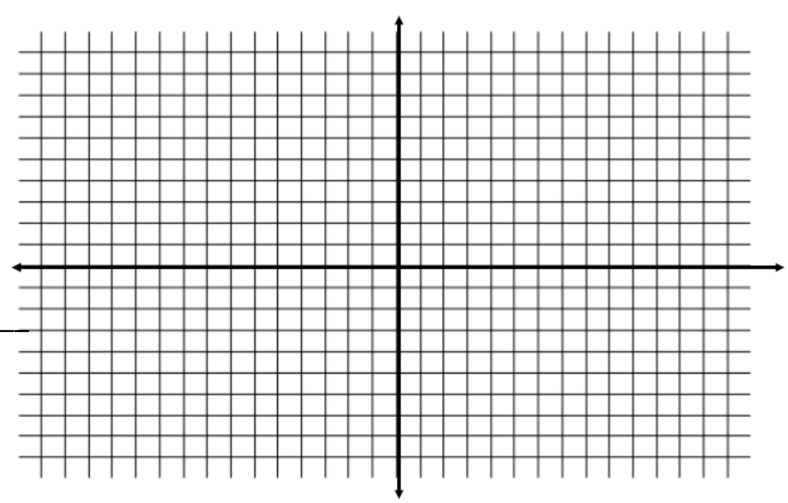
Examples: You may use your graphing calculator to help you. Answer the questions following each graph.

1.) Graph the function: $f(x) = x(x-1)(x+1)$

(a) What degree is this function? _____________

(b) What are the function's zeros?________________

(c) What are the solutions to f(x) = 0? _______________



(d) What are the x-intercepts of the graph?___________

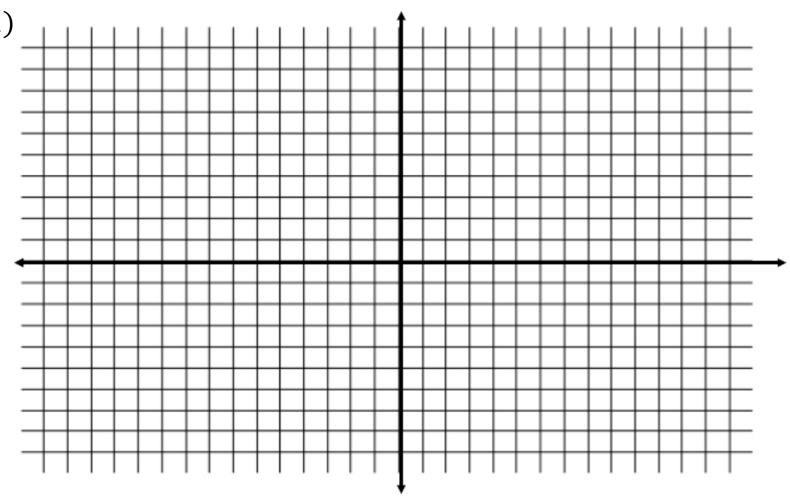
(e) What are the y-intercepts of the graph? ___________

(f) How does the degree compare to the x-intercepts? ____

____

(g) Describe the end behavior of the graph. ____

2.) Graph the function: $f(x) = (x+3)(x+3)(x-1)$



(a) What degree is this function?_____________

(b) What are the function's zeros?______________

(c) What are the y-intercepts? ______________

(d) State the end behavior: ____________________

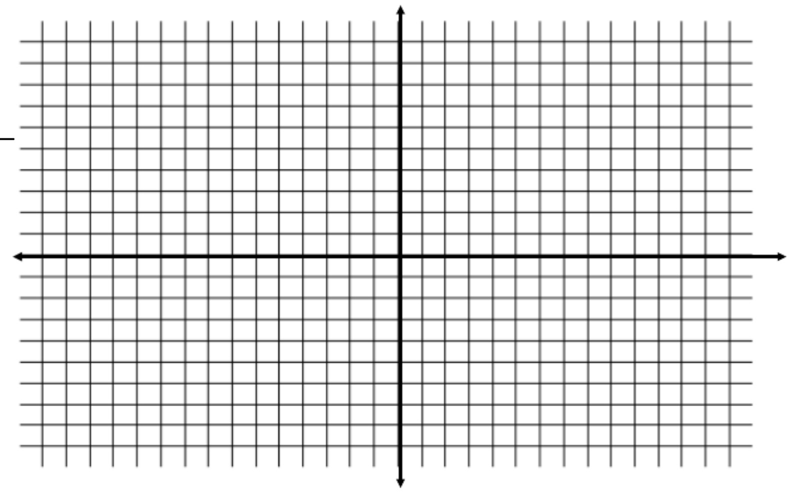
(e) What is special about this function?____

____

____

3.) Graph the function: $f(x) = (x^2 + 1)(x - 2)(x - 3)$

(a) What degree is this function ______________



(b) What are the x-intercepts of the graph? _________

(c) What are the y-intercepts? ______________

(d) What is the end behavior? ________________

(e) How does the degree compare to the x-intercepts? _____________________________

____

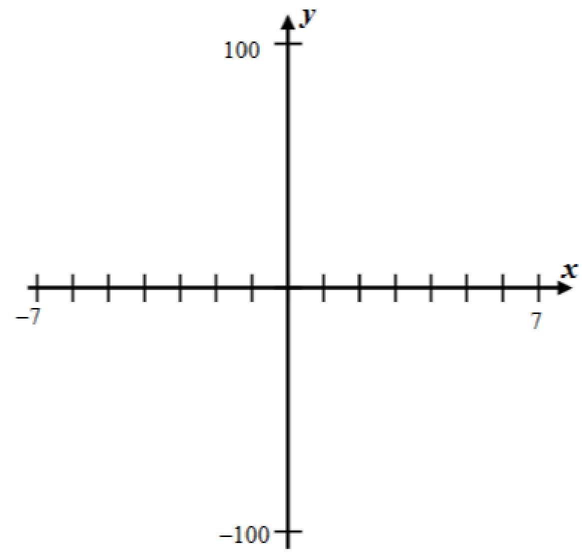
____

(g) Explain why the factor $x^2 + 1$ is never zero. ____________________________________

____

____

4.) Create the equation of the cubic, in standard form, that has x-intercepts of -4, 2, and 5 and goes through the point (6, 20). Then sketch the function.



Steps:

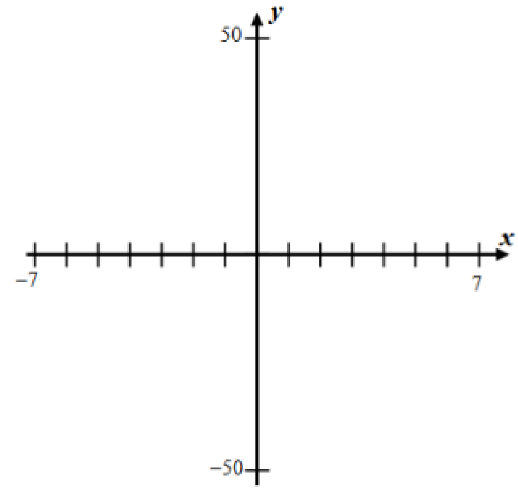
1.) Write the equation in factored form. Leave the coefficient as *a* in front, because we do not know whether the function has a negative or positive end behavior.

2.) Substitute the point (6,20) into the equation that you found.

3.) Solve for a.

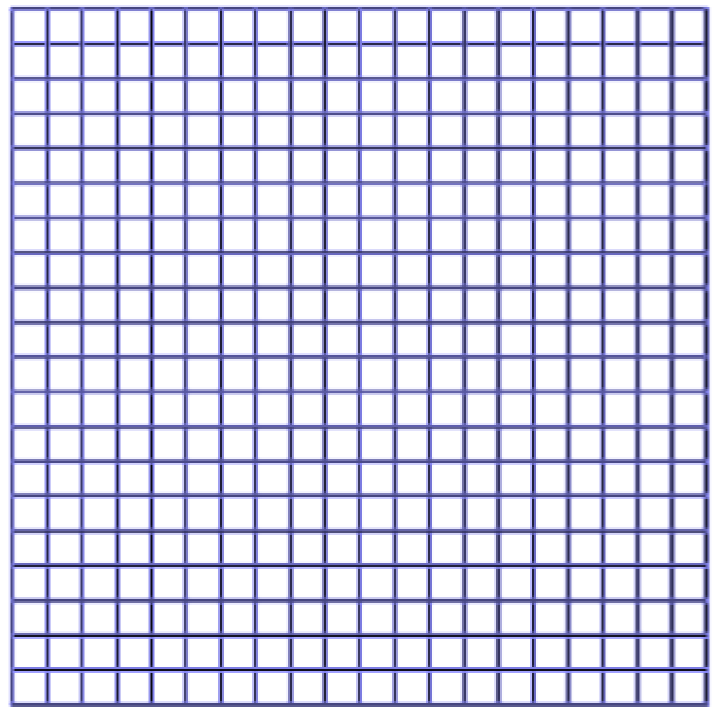
4.) Distribute.

5.) Create the equation of the cubic, in standard form, that has a ***double zero*** at -2 and another zero at 4. The cubic has a y-intercept of 4.



Chapter 8
Lesson 3: Graphing Polynomial Functions by Zeros
HOMEWORK

1.) Sketch a graph of the function $f(x) = -(x+2)(x-4)(x-1)$.

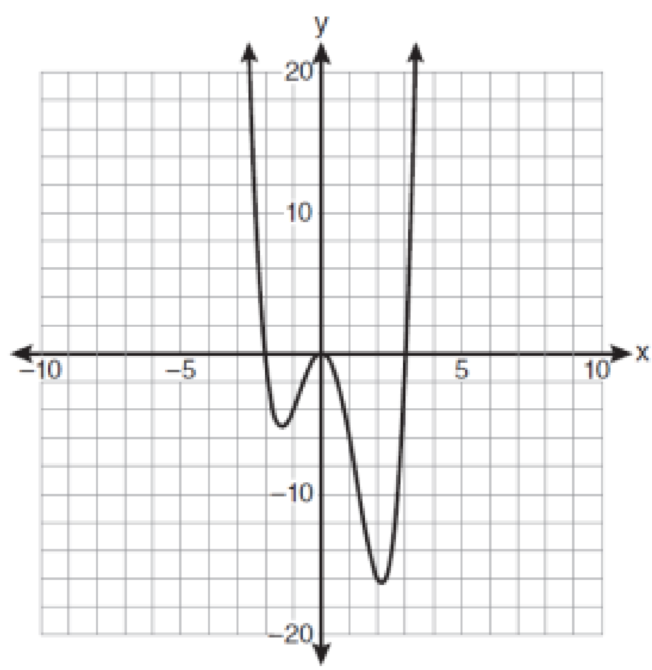


(a) What are the roots?

(b) Describe the end behavior of the graph.

(c) What is the y-intercept?

2.) The graph of y = f(x) is shown to the right.

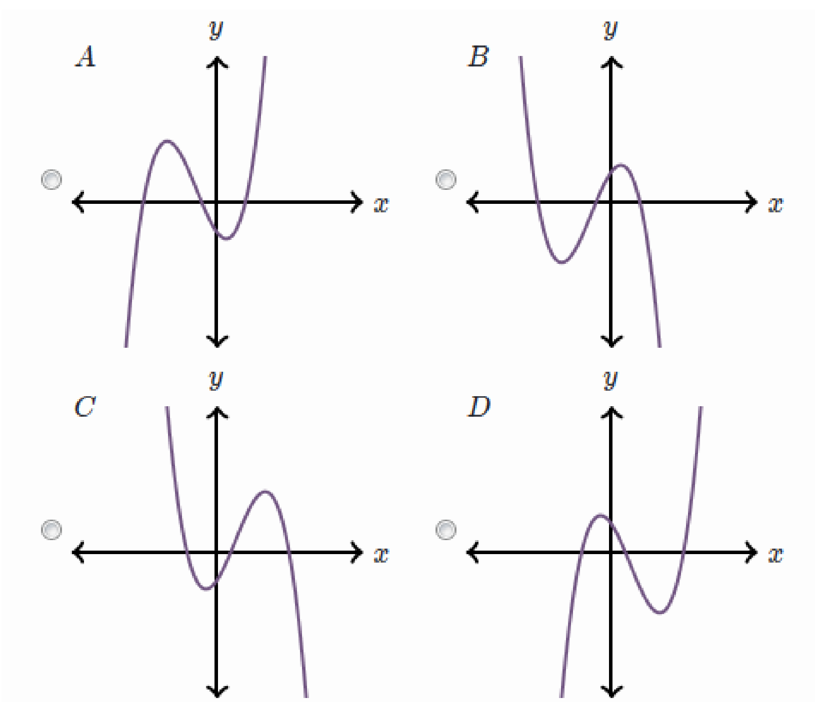


(a) Find all real solutions of f(x) = 0.

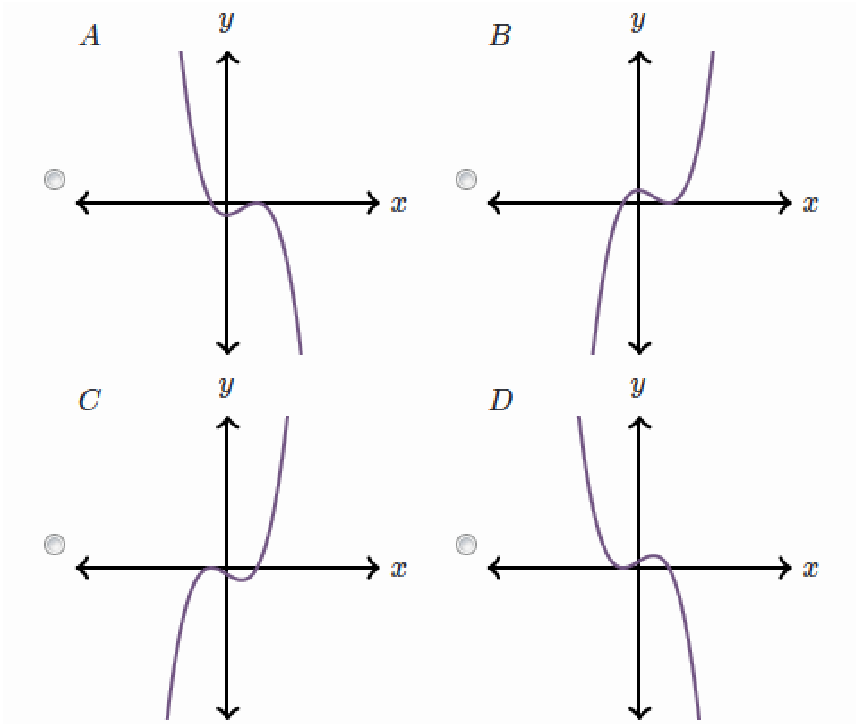
(b) Is there a double root? Justify your answer.

3.) Create the equation of a quadratic polynomial, in standard form, that has zeros of -5 and 2 and which passes through the point (3,-24). Sketch the graph of the quadratic.

4.) Which of the following graphs could be the graph of g(x) = (x + 1)(x – 2)(x + 5)?



5.) Which of the following could be the graph of $y = (2 - x)(x + 1)^2$?



Chapter 8

Lesson 4: Graphing Polynomial Equations

Graphs of Polynomials Not in Factored Form:

Recall: To Graph a Polynomial Function:

1) Find the functions' _______________ (roots)
2) State the ________________ (substitute 0 for x and solve)
3) State the _______________ behavior

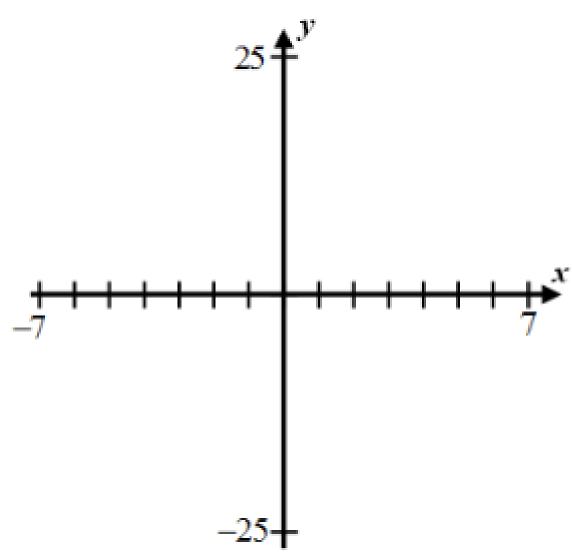
In order to graph: plot the roots and y intercepts. Draw in the end behavior. Connect the ends of the graphs by making sure you go through all intercepts. (Some graphs will be tangent to the x-axis at certain points. When this happens, the graph will not go through the x-intercept but rather hit the axis and turn around)

Examples:

1.) Consider the cubic whose equation $y = x^3 - x^2 - 12x$.

(a) *Algebraically* determine the zeros of this function.

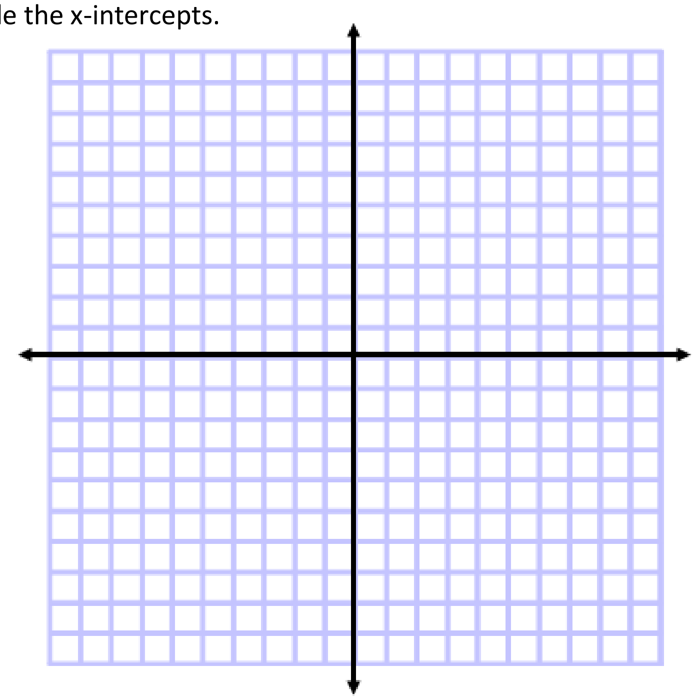
(b) Graph the polynomial on the grid provided.



2.) Consider the quartic $y = x^4 - 5x^2 + 4$.

(a) *Algebraically* determine the zeros of this function.

(b) Sketch the graph on the axes below. Circle the x-intercepts.



3.) The largest root of $x^3 - 9x^2 + 12x + 22 = 0$ falls between what two consecutive integers?

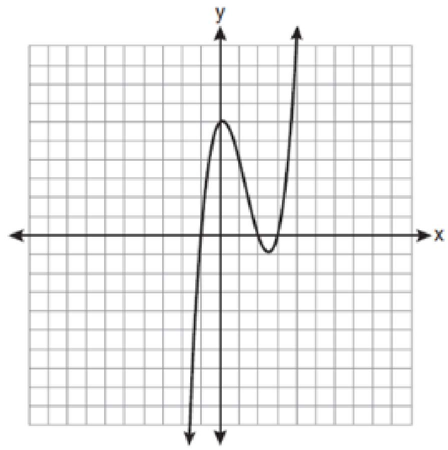
(1) 4 and 5

(2) 6 and 7

(3) 10 and 11

(4) 8 and 9

4.) The graph of $y = x^3 - 4x^2 + x + 6$ is shown on the graph below. What is the ***product*** of the roots of the equation $x^3 - 4x^2 + x + 6 = 0$?



(1) -36

(2) -6

(3) 6

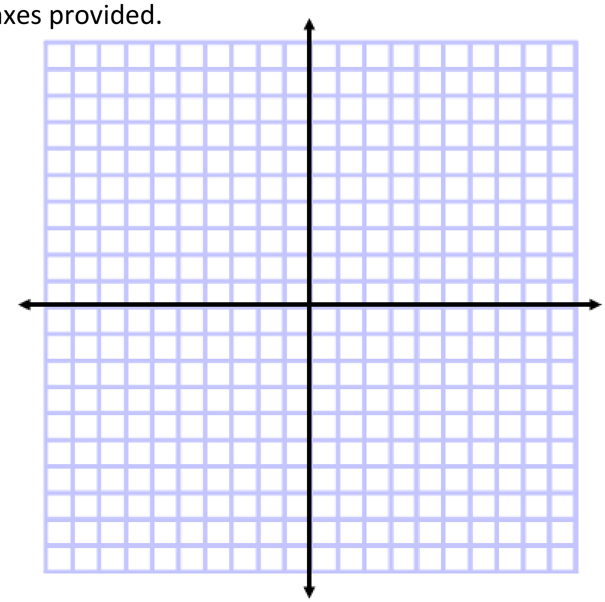
(4) 4

5.) Given the equation $f(x) = x^3 - 2x^2 - 3x$.

(a) Find the zeros of $f(x)$.

(b) What is the end behavior of this graph?

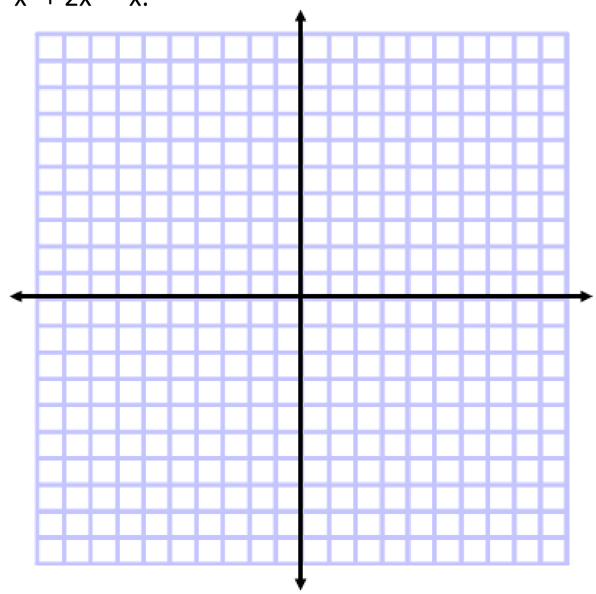
(c) Sketch the graph of $f(x)$ using the axes provided.



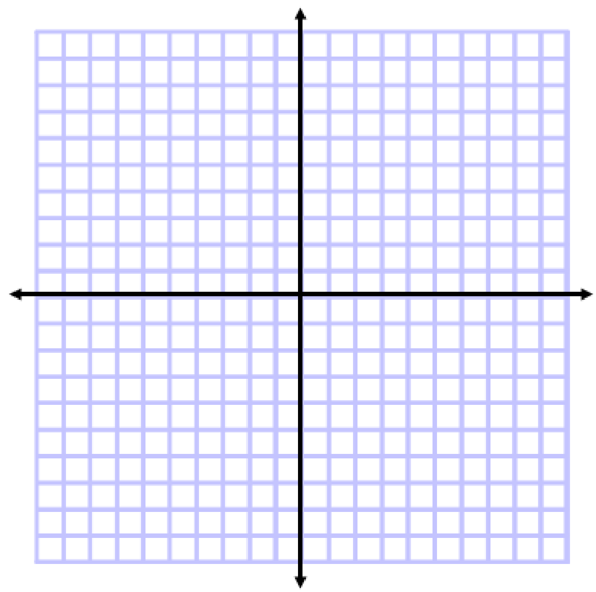
Chapter 8
Lesson 4: Graphing Polynomial Equations
HOMEWORK

Fluency:

1.) After factoring, sketch the graph of the equation $y = -x^3 + 2x^2 - x$.

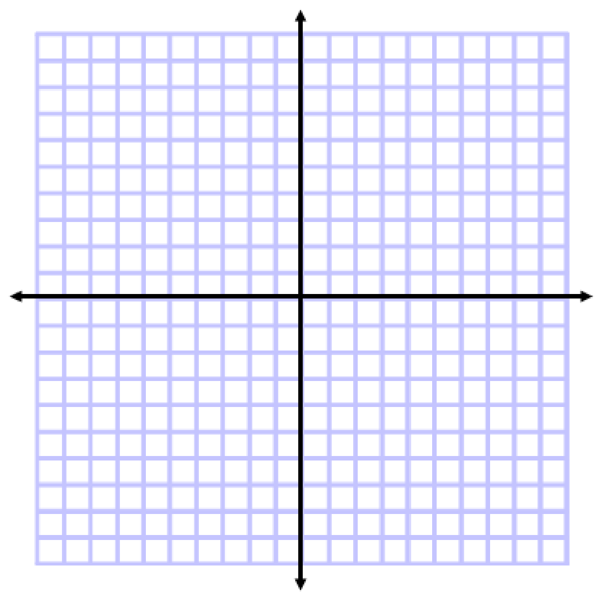


2.) After factoring, sketch the graph of the $g(x) = x^4 + x^3$.



3.) (a) Find *algebraically* the zeros for $p(x) = x^3 + x^2 - x - 1$.

(b) On the set of axes below graph y = p(x).



(c) Using your knowledge from a previous chapter, determine the coordinates of any relative extrema. Round to three correct decimal places. You will need your calculator to solve.

Chapter 8

Lesson 5: Polynomial Long Division

Sometimes we are asked to divide two polynomials. To do this, we use a process known as ***long division***.

Polynomial Long Division:

Divide: $x^2 - 9x - 10$ by $x + 2$

Steps:

1.) Write the expression in a form of a _______ ____________ problem: (if there are missing terms, you must use 0's as place holders).

2.) _________ the leading term of the numerator polynomial, by the leading term of the divisor and write the answer on the top line.

3.) _________ the answer on the top line to the expression outside the division bar, place those answers under their like terms under the division sign

4.) Subtract to create a new ________________.

5.) Repeat, using the new polynomial.

6.) If there is a remainder, turn it into a _________. The remainder is the numerator and the denominator is what you divided by.

When completing polynomial long division, it is important to be neat.

Examples: Complete the following problems by using polynomial long division.

1.) $(m^2 - 7m - 11) \div (m - 8)$

2.) $\frac{2x^2+15x+20}{x+6}$

3.) $(4x^2 - 8x + 6) \div (2x - 1)$

4.) $\frac{2x^3-11x^2+22x-25}{x-3}$

5.) $(4x^3 - 9x + 14) \div (x + 2)$

6.) Simplify $\frac{2x^2+15x+18}{x+6}$ by performing polynomial long division.

(a) Can we rewrite this as a multiplication problem? What is it?

(b) What does it mean when we have a remainder of zero?

Chapter 8
Lesson 5: Polynomial Long Division
HOMEWORK

Complete all questions below by polynomial long division.

1.) $\frac{2x^2-23x+17}{x-10}$

2.) $\frac{5x^2-41x+3}{x-8}$

3.) $\frac{x^3+7x^2+17x+41}{x+5}$

4.) $\frac{6x^2+11x-10}{3x-2}$

5. $(5x^3 + 6x - 18) \div (x + 2)$

6.) If the ratio $\frac{2x^2+17x+42}{x+5}$ is placed in the form $q(x) + \frac{r}{x+5}$, where q(x) is a polynomial, then which of the following is the correct value of *r*?

(1) -3

(3) 18

(2) 177

(4) 7

Chapter 8
Lesson 6: Remainder Theorem

Do Now:

1.) Consider each of the following scenarios where we have $\frac{p(x)}{x-a}$. In each case, simplify the division using polynomial long division and then evaluate *p*(*a*).

(a) $\frac{x^2-8x+18}{x-2}$ p(x) = $x^2 - 8x + 18$; Evaluate: p(2)

(b) $\frac{2x^2+11x+11}{x+3}$ p(x) = $2x^2 + 11x + 11$; Evaluate: p(-3)

(c) What do you notice about the remainder of the long division problem and the value of p(a)?

Polynomial Remainder Theorem:

When we divide a polynomial f(x) by (x – c), the remainder *r* equals ________________.

Examples:

1.) If the ratio $\frac{x^2-11x+22}{x-9}$ was placed in the form $q(x)+\frac{r}{x-9}$ where *q*(*x*) is a linear function, then which of the following is the value of *r*?

(1) -3 (3) 9

(2) 5 (4) 4

2.) What is the remainder when you divide $2x^2 - 5x - 1$ by $x - 3$?

3.) What is the remainder when $2x^2 - 5x + 12$ is divided by $2x + 1$?

4.) Find the remainder when $2x^3 - x^2 - x - 1$ is divided by $x - 1$.

5.) Factor: $x^2 - 3x - 4$, then find the remainder when $x^2 - 3x - 4$ is divided by $x - 4$. What do you notice?

Factor Theorem:

When we divide a polynomial $f(x)$ by $(x – c)$ and the remainder is _____________, then $(x – c)$ __________________________ of the polynomial!

Examples:

6.) Use the Factor Theorem to determine whether $x – 1$ is a factor of $f(x) = 2x^4 + 3x^2 – 5x + 7$.

7.) Is $x + 5$ a factor of $p(x) = 2x^2 + 9x – 5$?

Chapter 8
Lesson 6: Remainder Theorem
HOMEWORK

1.) Which of the following linear expressions is a factor of the cubic polynomial $x^3 + 9x^2 + 16x - 12$?

(1) x + 6 (3) x – 3

(2) x – 1 (4) x + 2

2.) When the polynomial p(x) was divided by the factor x – 7 the result was $x + \frac{11}{x-7}$. Which of the following is the value of p(7)?

(1) -8 (3) 11

(2) 7 (4) It does not exist

3.) Find the remainder when the polynomial $x^2 - 5x + 3$ is divided by the binomial (x – 8).

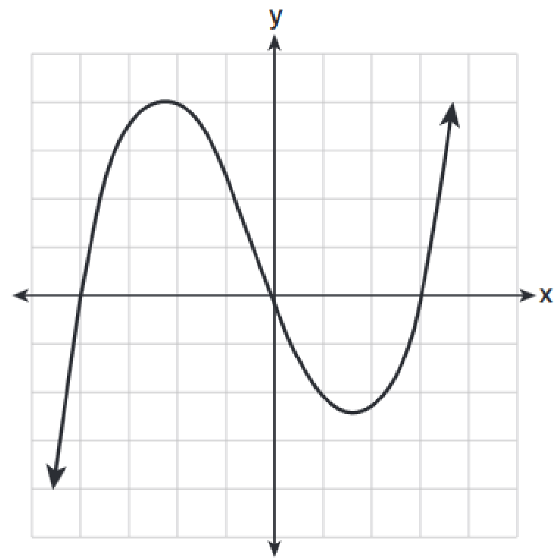
4.) The graph of p(x) is shown below. What is the remainder when p(x) is divided by x + 4?

(1) x – 4

(2) -4

(3) 0

(4) 4



5.) Determine if (x – 2) is a factor of $P(x) = x^3 - 3x^2 + 5x - 2$. Explain your answer.

6.) Is 3 a root in the polynomial expression $2x^3 - x^2 - 7x + 2$? Use the factor theorem to justify your answer.