Chapter 10: Arithmetic & Geometric Sequences

A: Introduction to Sequences – U4L12

B: Recursive Sequences

C: Arithmetic Sequences – U4L13

D: Geometric Sequences -U6L9

Name:____ Date:__________________

Algebra I – Pd ___ Introduction to Sequences 10A - Classwork

Sequence Definition

A sequence is a function whose set of inputs, the domain, is a subset of the natural numbers, i.e. $\{1, 2, 3, 4 \ldots\}$ A sequence is often shown as an ordered list of numbers, called the terms or elements of the sequence. Sequence function notation can be tricky.

Given the sequence {2, 4, 6, 8, …}, the first term can either be noted as $a(1) = 2$ or $a_1 = 2$. The letter does not have to be a either, it can be written as $f_3 = 6$ which means the 3rd term is 6 or $b(2) = 4$, meaning the 2nd term is 4. A **sequence** is formally defined as **a function that has as its domain the set of positive integers**, i.e. $\{1, 2, 3, \ldots, n\}$.

The notation $a(n)$ or a_n is referring the n^{th} term, which is often used to refer to the last term or used in a formula.

Finally, the notation $a(n-1)$ or a_{n-1} means the term before the n^{th} term; or the previous term.

Exercise #1: Consider the sequence below. If we represent this sequence with the letter a, then:

$$4, 8, 16, 32, 64, 128, 256$$

(a) Find $a(3)$

(b) Find $a(1) + a(7)$

(c) Find a_2

(d) Find $(a_1)^2$

(e) Find $a_5 - a_4$

(f) Solve for n: $a(n) = 128$

Sequences are functions. The key here is that the input is simply the number's place in line so to speak and the output is the actual number in the list.

Exercise #2: Consider the sequence defined by the formula $a(n) = 2n + 1$

(a) Write out the first five elements of this sequence. The first term has been found for you.

$a(n) = 2n + 1$
$a(1) = 2(1) + 1$
$a(1) = 3$

The first term is 3

(b) What is the 21st term of this sequence? Show how you arrived at your answer.

Exercise #3: Find the first three terms in each sequence.

(a) $a_n = 2n^2 + 5$

(b) $a(n) = \frac{2n+1}{n}$

Exercise #4: Given the following sequences, find the next three terms and the specific term listed.

(a) $35, 32, 29, 26,$_____, _____, _____

$a(10) =$ _____

Explain how you found $a(10)$.

(b) $-3, 1, 5, 9,$ _____, _____, _____

$a_{12} =$ _____

How would you find a_{120}?

What about a_{600}, how would you find this?

The NYS Reference sheet gives you the following Arithmetic Sequence formula. This formula can be used to find any term in a sequence.

Arithmetic Sequence: $\boldsymbol{a_n = a_1 + (n-1)d}$

Where $a_n = n^{th}\ term$; $a_1 = 1^{st}\ term$

n = the term number that you are looking for; d = common difference

Try to redo a_{120} and a_{600} from Exercise #4 using the formula.

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Algebra I CC – Pd ___ Introduction to Sequences 10A Homework

1.) Consider the sequence below. If we represent this sequence with the letter a, then do the following:

$$1, 7, 13, 19, 25, 31, 37, 43$$

(a) Find $a(5)$

(b) Find $a_2 + a_6$

(c) Find $a(4) + 2a(6)$

(d) Find $\sqrt{a(5)}$

(e) Find $\frac{a(5) - a(3)}{2}$

2.) Consider the sequence defined in the table below.

n	1	2	3	4	5
$b(n)$	2	12	22	32	42

(a) Find $b(4)$

(b) Find $\frac{2b(2) - b(3)}{4}$

3.) Consider a sequence of numbers given by the definition and $c_i = 2(i)^2 + 3$

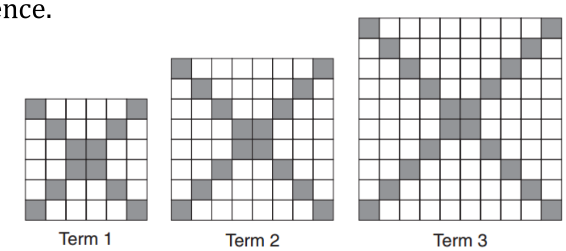
(a) Write out the first 4 terms of this sequence.

(b) Find the value of $c_4 - c_2$. Show calculations.

4) Find the fifth term in a sequence where the first term is 4 and the common difference is 3.

____6.) The diagrams below represent the first three terms of a sequence. Assuming the pattern continues, which formula determines a_n the number of shaded squares in the nth term?

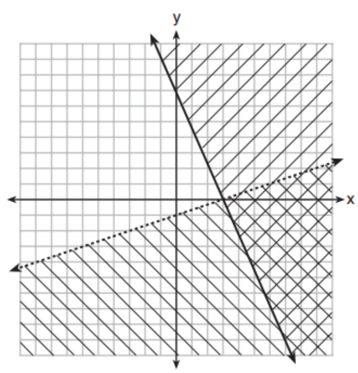
[1] $a_n = 4n + 12$ [2] $a_n = 4n + 8$
[3] $a_n = 4n + 4$ [4] $a_n = 4n + 2$



Review Section:

____7.) Which statement is not always true?
[1] The product of two irrational numbers is irrational.
[2] The product of two rational numbers is rational.
[3] The sum of two rational numbers is rational.
[4] The sum of a rational number and an irrational number is irrational.

____8.) What is one point that lies in the solution set of the system of inequalities graphed below?
[1] (7,0)
[2] (0,7)
[3] (3,0)
[4] (-3,5)



10A Homework Answers

1.) a.) 25 b.) 38 c.) 81
d.) 5 e.) 6

2.) a.) 32 b.) $\frac{1}{2}$

3.) a.) $\{5, 11, 21, 35\}$ b.) 24

4.) 16

6.) [2]

7.) [1]

8.) [1]

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Algebra I – Pd ___ Recursive Sequences 10B - Classwork

Sequences can be defined explicitly, by a classic function formula, like what we saw in lesson A, and they also can be defined **recursively**. A **recursive formula** is one where each term in the sequence **depends on a term or terms** that came **before it**.

Recall: The notation $a(n-1)$ or a_{n-1} means the term before the n^{th} term; or the previous term.

Exercise #1: Consider a sequence of numbers given by the following definition:

$$b_1 = 7 \quad and \quad b_n = b_{n-1} + 4$$

(a) Give a common sense interpretation for this **recursive** function rule.

(b) Write out the rule for the first 4 terms and evaluate each one of them (except b_1).

Exercise #2: A sequence is defined by the rule: $f(n) = 3f(n-1) + 4; if\ f(1) = 2$, find $f(2) and\ f(3)$.

One of the most famous of all **recursively defined** sequences is known as the **Fibonacci Sequence**. Let's play around with it in the next exercise.

Exercise #3: The Fibonacci Sequence is defined recursively as follows:

$$a(1) = 1, \quad a(2) = 1, \quad and \quad a(n) = a(n-1) + a(n-2)$$

(a) How do you interpret this recursive rule? Write it down in your own words.

(b) Write down the rule for $a(3), a(4)$ and $a(5)$ and determine their values.

Sequences often show up in the real world, where they are defined in terms of a recursive process.

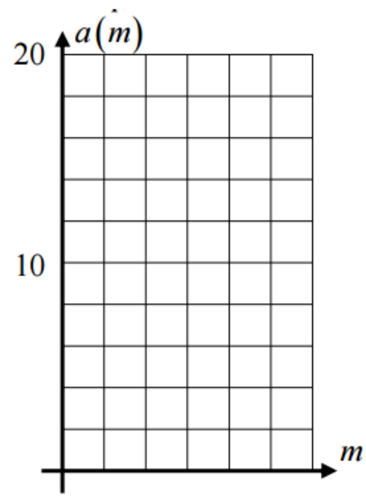
Exercise #4: Kirk is trying to train for the marathon. His first month, he runs 5 miles per workout. He adds an additional 3 miles to his workout for each month that he trains.

(a) Fill out the table below for the amount of miles he runs as a function of how many months he has been running.

m	1	2	3	4	5
$a(m)$	5				

(b) Give a recursive definition for the sequence $a(m)$ Don't forget to give an initial value.

(c) Graph this sequence for $1 \leq m \leq 5$.



Exercise #5: Is the formula $a_n = -4n(n-1)$ *explicit* or *recursive*? Find the first five terms of the sequence

Exercise #6: A sequence is defined by the recursive formula $a_1 = -6$ and $a_n = a_{n-1} + 2n$.

Write out the next three terms of this sequence. Show the work that leads to your terms.

Exercise #7: The third term in an arithmetic sequence is 10 and the fifth term is 26. If the first term is $a(1)$, which is an equation for the *nth* term of this sequence?

(1) $a(n) = 8n + 10$
(2) $a(n) = 8n - 14$
(3) $a(n) = 16n - 38$
(4) $a(n) = 16n - 38$

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Algebra I – Pd ____ Recursive Sequences 10B - Homework

1.) Consider the sequence below. If we represent this sequence with the letter a, then do the following:

$$1, 7, 13, 19, 25, 31, 37, 43$$

Find a recursive definition for the sequence $a(n)$; remember to state the initial value

2) Consider the sequence defined in the table below.

n	1	2	3	4	5
$b(n)$	2	12	22	32	42

Find a recursive definition for the sequence b_n remember to state the initial value.

3.) Consider a sequence of numbers given by the definition $c_1 = 2$ and $c_i = c_{i-1} \cdot 3$
(a) Write out the first 4 terms of this sequence.
(b) Find the value of $c_4 - c_2$. Show calculations.

4) If $f(1) = 3$ and $f(n) = -2f(n-1) + 1$, then find $f(5)$.

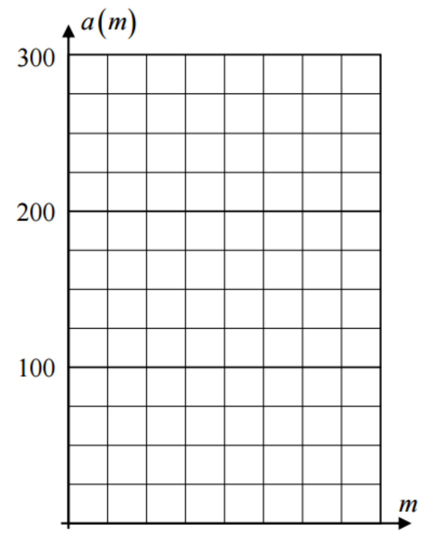
5.) Erin is traveling abroad this summer and would like to have a bit of spending cash while she's overseas. She has 100 dollars already saved and she plans on saving 40 dollars a month.

(a) Fill out the table below for the amount of money she saves as a function of how many months she has been saving.

m	1	2	3	4	5
$a(m)$	140				

(b) Give a recursive definition for the sequence $a(m)$. Don't forget to give an initial value.

(c) Graph this sequence for $1 \leq m \leq 5$



6.) A sequence is defined recursively as follows: $a(1) = 1$ and $a(n) = a(n-1) + n$

(a) How do you interpret this recursive rule? Write it down in your own words.

(b) Write down the rule for $a(2)$, $a(3)$ and $a(4)$ and determine their values.

____7.) A sunflower is 3 inches tall at week 0 and grows 2 inches each week. Which function(s) shown below can be used to determine the height, $f(n)$, of the sunflower in n weeks?

I. $f(n) = 2n + 3$

II. $f(n) = 2n + 3(n-1)$

III. $f(n) = f(n-1) + 2$ where $f(0) = 3$

[1] I and II [2] III, only

[3] II, only [4] I and III

REVIEW SECTION:

_____8.) The owner of a small computer repair business has one employee, computations. who is paid an hourly rate of \$22. The owner estimates his weekly profit using the function P(x) = 8600 - 22x. In this function, x represents the number of

[1] computers repaired per week
[2] hours worked per week
[3] customers served per week
[4] days worked per week

8B Homework Answers:

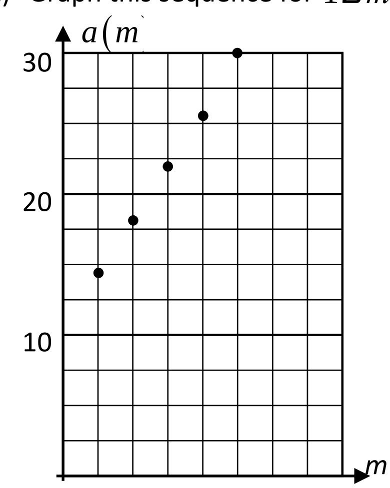
1) $a_n = a_{n-1} + 4;\ a_1 = 1$
2) $a(n) = a(n-1) + 10; a(1) = 2$
3.) a.) $\{2, 6, 18, 54\}$ b.) 48
4) 43

5.) a.)

m	1	2	3	4	5
$a(m)$	140	180	220	260	300

b.) $a_m = a_{m-1} + 40;\ a_1 = 140$ c.)

(c) Graph this sequence for 1£m£5.



6.) a.) The first term in the sequence is 1. The other terms are found by adding the previous term to its place in line, n.

b.) $\{3, 6, 10\}$

7) (4)

8) (2)

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Algebra I CC – Pd____ Arithmetic Sequences 10C - Classwork

There are many types of sequences, but there is one that is related to linear functions and in fact is a type of **discrete linear function**. These are known as **arithmetic sequences**. Let's illustrate one first.

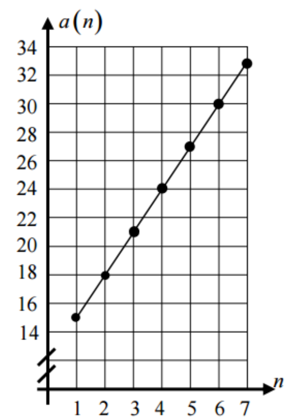
Exercise #1: Erin is saving money for to buy a new toy. She already has $12 in her account. She gets an allowance of $4 per week and plans to save $3 in her account.

(a) Fill out the table below for the amount of money Erin has after n weeks of saving?

n	$a(n)$
1	$15
2	$18
3	
4	
5	
6	
7	

(b) Write a recursive definition for this sequence.

(c) What's wrong with the graph of the sequence shown below?



(d) Erin proposes the following explicit formula for the amount of savings, a, as a function of the number of weeks saved, n. Is the formula correct? Test it!

$a(n) = 3n + 15$

Arithmetic sequences are ones where the **terms** in the **list** increase or decrease by the same amount given a unit increase in the **index** (where the number is in line).

Exercise #2: An arithmetic sequence is given using the recursive definition: $b_1 = -3$ and $b_i = b_{i-1} + 6$. Which of the following is the value of b_4? Show the work that leads to your answer.

[1] 24 [2] 12

[3] 21 [4] 15

Arithmetic sequences are relatively easy to spot and are easy to fill in, so to speak.

Exercise #3: For each of the following sequences, determine if it is arithmetic based on the information given. If it is arithmetic, fill in the missing blank. If it is not, show why.

(a) 5, 9, 13, __________, 21, 25

(b) 5, 10, 20, 40, __________, 160

(c) 7, 4, 1, __________, $-5,\ -8$

(d) 64, 16, 4, __________, $\frac{1}{4},\ \frac{1}{16}$

Finding a specific term in an arithmetic sequence without listing them sometimes can be a challenge, but not if you take your time and really think about it.

Exercise #4: Consider an arithmetic sequence whose first three terms are given by: 4, 14, 24

(a) What is the 4th term? How many times was 10 added to 4 to get to the 4th term? Show a diagram to illustrate this.

(b) Use what you learned in part (a) to find the value of a_{10}, 10th the term.

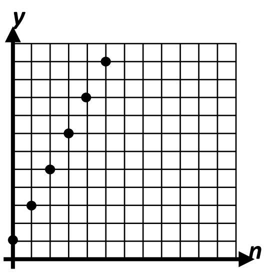
(c) Write a recursive formula for the a_n based on the term number n.

(d) Write an explicit formula for a_n

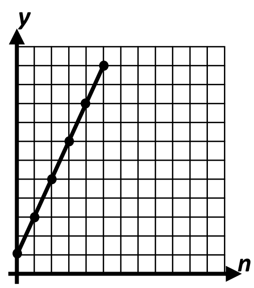
Exercise #5: Seats in a small amphitheater follow a pattern where each row has a set number of seats more than the last row. If the first row has 6 seats and the fourth row has 18, how many seats does the last row, which is the 20th, have in it? Show your work to justify your response.

______***Exercise #6:*** Which of the following would represent the graph of the sequence $a_n = 2n + 1$? Explain your choice.

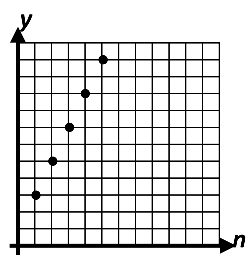
(1)



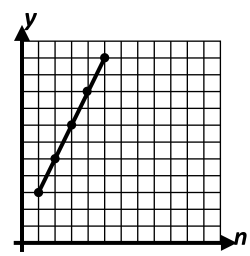
(2)



(3)



(4)



Explanation:

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Algebra I CC – Pd____ Arithmetic Sequences 10C - Homework

____1.) An arithmetic sequence is given using the recursive definition: $b_1 = 8$ and $b_i = b_{i-1} - 2$. Which of the following is the value of b_4? Show the work that leads to your answer.

[1] 14 [2] 2
[3] 6 [4] 4

2.) For each of the following sequences, determine if it is arithmetic based on the information given. If it is arithmetic, fill in the missing blank. If it is not, show why.

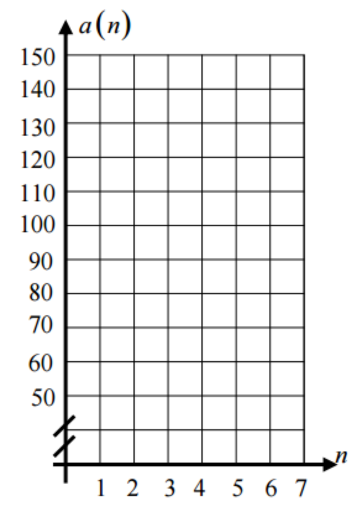
(a) 12, 24, 36, ____ , 60, 72

(b) 10000, 1000, ______, 10, 1

(c) ____ , 24, 20, 16, 12, 8

(d) $\frac{1}{4}$, $\frac{1}{2}$, _____, 1, $\frac{5}{4}$

3.) Given the sequence defined by the explicit formula $g(n) = 15n + 35$ write out the first four terms. Then, create a recursive definition and graph the sequence on the interval $1 \leq n \leq 7$.



____4.) Which of the following is an arithmetic sequence?

[1] 2, 4, 8, 16, 32, 64 [2] 50, 45, 40, 35, 30

[3] 1, 1, 2, 3, 5, 8, 13 [4] 1, $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{16}$

5.) Mike is building a tower out of paper cups. In each row (counting from the floor up), there are two less cups than the row below it. The first row has 26 cups in it.

(a) State the number of cups in the second, third, and fourth rows.

(b) Give a recursive definition for this arithmetic sequence.

(c) How many cups will be in the 11th row? Show the calculation that leads to your answer.

6.) Eric considers the sequence of numbers given by the following definition: $b_1 = 7$ and $b_i = b_{i-1} + 4$ and decides the first 4 numbers are: 4, 11, 18, 25

(a) Interpret in your own words, what the sequence is saying and what he actually did.

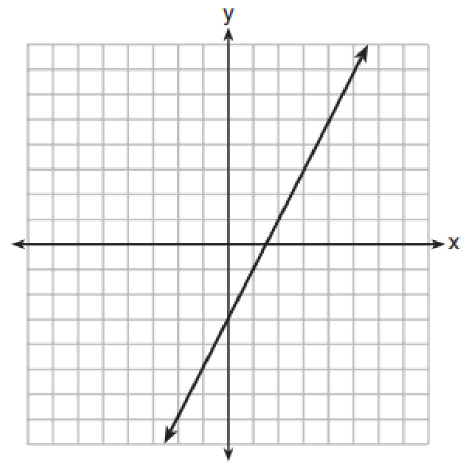
(b) What should the first four numbers be?

Review Section:

_____7.) Which domain would be the most appropriate set to use for a function that predicts the number of household online-devices in terms of the number of people in the household?

[1] integers [2] irrational numbers
[3] whole numbers [4] rational numbers

_____8.) Which function has the same y −intercept as the graph below?



[1] $y = \frac{12-6x}{4}$
[2] $27 + 3y = 6x$
[3] $6y + x = 18$
[4] $y + 3 = 6x$

_____9.) Fred is given a rectangular piece of paper. If the length of Fred's piece of paper is represented by $2x - 6$ and the width is represented by $3x - 5$, then the paper has a total area represented by:

[1] $5x - 11$ [2] $6x^2 - 28x + 30$
[3] $10x - 22$ {4} $6x^2 - 6x - 11$

_____10.) The graph of a linear equation contains the points $(3,11)$ and $(-2,1)$. Which point also lies on the graph?

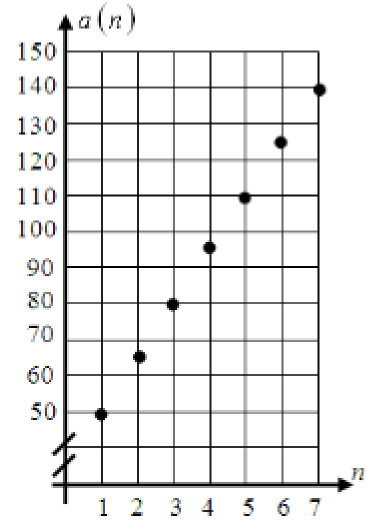
[1] (2,1) [2] (2,4)
[3] (2,6) [4] (2,9)

10C Homework Answers

1.) [2]

2.) a.) yes; 48 b.) no
c.) yes; 28 d.) yes, $\frac{3}{4}$

3.) $\{50, 65, 80, 95, 110, 125, 140\}$ $g_n = g_{n-1} + 15;\ g_1 = 50$



4.) [2]

1) 5.) a.) $\{24, 22, 20\}$ b.) $c_n = c_{n-1} - 2;\ c_1 = 26$

c.) 6 cups

6.) a.) The sequence definition is saying that the first term is 7 and each term is 4 more than the previous. What Eric did was have a first term of 4 and then add 7 to the previous term.
b.) $\{7, 11, 15, 19\}$

7.) [3]

8.) [4]

9.) [2]

10.) [4]

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Algebra I - Pd ____ Geometric Sequences 10D- Classwork

We just looked at **sequences,** which just consisted of a specific **list of numbers** in a **particular order**. We extensively studied the idea of an **arithmetic sequence**, where each successive number in the list was generated by adding the same quantity to the previous number. Let's do a warm up.

***Exercise #1*:** An arithmetic sequence is defined **recursively** by the following formula:

$$a_1 = 5 \text{ and } a_n = a_{n-1} + 3$$

(a) Find the next three terms of the sequence.

(b) Find the value of a_{20} without listing out all 20 terms.

Clearly, **arithmetic sequences** share many characteristics of **linear functions.** In fact, **arithmetic sequences** are examples of **discrete linear functions**. **Exponential functions** have their own **discrete versions** and those are called **geometric sequences**. They have a very simple **recursive definition**.

GEOMETRIC SEQUENCES

Given the first term, a_1, then each successive term can be found by $a_n = a_{n-1} \cdot r$, where r is some constant often known as the **common ratio** of the sequence.

The common ratio can be found by dividing any term by its previous term. For example: $r = \frac{a_2}{a_1}$

***Exercise #2*:** For each of the following **geometric sequences** identify the common ratio, *r*, and give the next two terms.

(a) 2, 6, 18, _______, _______

$r =$

(b) $4, -20, 100$, _______, _______

$r =$

(c) 16, 8, 4, _______, _______

$r =$

As with arithmetic sequences, we should be able to predict any particular **term** in the geometric sequence by thinking about how many times we have multiplied by the **common ratio**, *r*.

***Exercise #3*:** Consider the geometric sequence given by the **recursive rule:**

$$b(1) = 3 \; and \; b(n) = b(n-1) \cdot 2$$

(a) Find $b(2), b(3), and \; b(4)$. Write each as extended product to see a pattern, but also find the final result.

(b) Based on (a), determine the value of $b(10)$ and $b(20)$.

(c) Based on the geometric sequence given, what would you do to find the 80th term? (Do not solve for this, just explain how you would find it.)

(d) The NYS Regents also gives a formula to find the *nth* term of a geometric sequence. Use this formula to find the 80th term.

Geometric Sequence: $a_n = a_1 r^{n-1}$

Where $a_n = n^{th} \; term$; $a_1 = 1^{st} \; term$

n = the term number that you are looking for; r = common ratio

What did the calculator give you? Let's talk about this number.

One thing you might have noticed in the last exercise is how quickly a geometric sequence grows. Does this sound familiar? Let's take a look at a classic problem.

***Exercise #4*:** You have just won a very strange lottery. The lottery promises to give you money each day for a 30 day month based on one of two options:

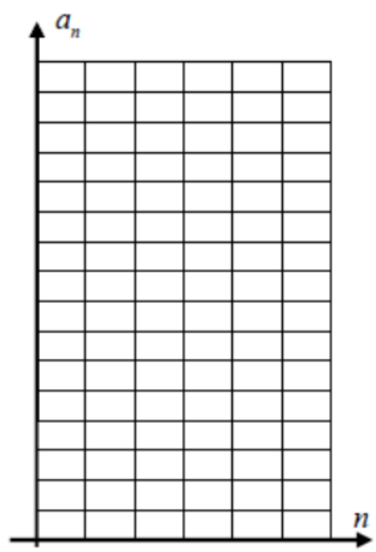
Option 1: You can receive $1000 on the first day, $2000 on the second day, $3000 on the third, in this **arithmetic sequence**.

Option 2: You can receive $0.01 (one penny) on the first day, $0.02 on the second, $0.04 on the third, $0.08 on the fourth, etcetera in this **geometric sequence**.

Of the two options, which would result in the larger payoff on the 30th day only? Show work that supports your answer.

Graphs of geometric sequences will look familiar. Because they are a type of **discrete exponential function** they will look very similar.

***Exercise #5*:** For a geometric sequence defined by $a_1 = 16$ and $a_n = a_{n-1} \cdot \frac{1}{2}$, list and plot the first 6 terms on the grid below.



Name:____ Date:_____________________

Algebra I - Pd ____ Geometric Sequences 10D - Homework

1. For each of the following geometric sequences, fill in the missing two terms and identify the common ratio, r. Remember, you can always find r by dividing two consecutive terms such as $\frac{a_2}{a_1}$ or $\frac{a_7}{a_6}$, dividing any consecutive terms will find r.

(a) 2, 10, 50, _______ , _______

r = ________

(b) 4, -8 , 16, _______ , _______

r = ________

(c) 40, 20, 10, _______ , _______

r = ________

(d) 81, 54, 36, _______ , _______

r = ________

(e) 5, -5 , 5, _______ , _______

r = ________

(f) 8, 20, 50, _______ , _______

r = ________

2. One of the following sequences is arithmetic and one is geometric. Explain which is which.

Sequence #1: 5, 15, 45, 135, 405

Sequence #2: 5, 15, 25, 35, 45

_____3. In a geometric sequence the first term is 5 and the second term is 20, which of the following is the fifth term?

(1) 65 (3) 80

(2) 1,280 (4) 5,120

4. A geometric sequence is defined recursively by $a(1) = 40$ and $a(n) = a(n-1) \cdot \frac{1}{2}$

(a) Write out the first four terms of this sequence.

(b) Is the 9th term of this sequence larger or smaller than $\frac{1}{10}$? Show the calculation that you use to determine your answer.

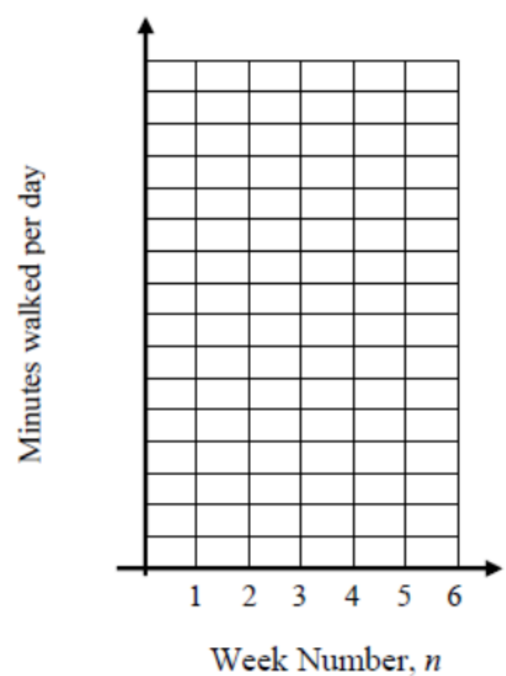
5. Which has the larger 15th term when comparing the arithmetic and geometric sequences below? Show evidence that supports your answer.

Arithmetic Sequence: 150, 650, 1150, 1650, …

Geometric Sequence: 4, 12, 36, 108, …

6. Maria plans to double the amount of time she spends walking per day each week. She starts, on week 1, walking 5 minutes per day. After 7 days, she then walks 10 minutes per day, etctera.

(a) How many minutes per day will Maria be walking on Week #6? Show the calculation that gives your answer.



(b) Scale the *y*-axis appropriately and graph the first six terms of this sequence. List them all if you haven't already.

(c) According to this geometric progression, how many minutes per day would Maria be walking on Week #10? Why is this not a viable answer?

Review Section:

______7) Which of the following represents the average rate of change of the function $h(x) = \frac{3}{2}x + 1$ over the interval $-2 \leq x \leq 8$?

(1) $\frac{9}{7}$ (3) $\frac{2}{3}$

(2) $\frac{5}{4}$ (4) $\frac{3}{2}$

8) Does the point (5, 4) lie in the solution set of the inequality $y \geq 2x - 4$? Justify your answer.

10D Homework Answers

1) (a) $r = 5; \quad 250, 1250$ (b) $r = -2; \quad -32, 64$ (c) $r = \frac{1}{2}; \quad 5, \frac{5}{2}$

(d) $r = \frac{2}{3}; \quad 24, 16$ (e) $r = -1; \quad -5, 5$ (f) $r = \frac{5}{2}; \quad 125, \frac{625}{2}$

2) Sequence #2 is increasing at a constant rate making it Arithmetic. It has a common difference of 10. Sequence #1 is increasing by a factor of 3 for each term, making this Geometric.

3) (2)

4) (a) $\{40, 20, 10, 5\}$

(b) $a(9) = 0.15625$ which is greater than 0.10, so $a(9)$ is larger than $\frac{1}{10}$.

5) Arithmetic: $a(15) = 7{,}150$

Geometric: $a(15) = 19{,}131{,}876$

The Geometric has a larger 15^{th} term. The Geometric Sequence grow (or decay) at a faster rate than Arithmetic Sequences because Arithmetic grow linearly, while Geometric grow exponentially.

6) (a) $160\ minutes$

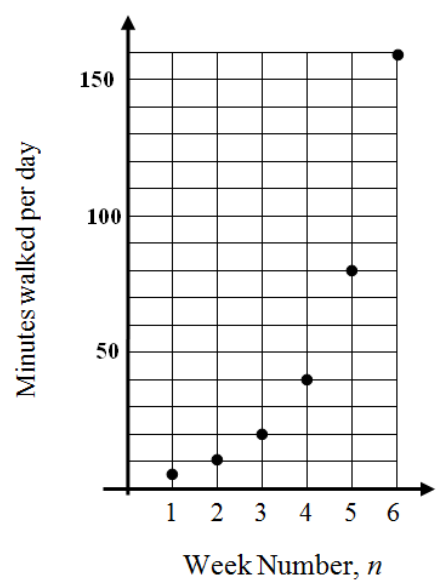
(b) $a_1 = 5 \qquad a_4 = 40$

$a_2 = 10 \qquad a_5 = 80$

$a_3 = 20 \qquad a_6 = 160$

(c) $a_{10} = 2{,}560\ minutes$

Since there are only 1,440 minutes in a day, this is not viable



7) (4)

8) No, it does not4 is not greater than or equal to 6.