ALGEBRA II (COMMON CORE)

FACTS YOU MUST KNOW COLD
FOR THE REGENTS EXAM
Algebra II [Common Core] Regents Exam Study Guide

**ALGEBRA & FUNCTIONS**

**FACTORING**

The Order of Factoring:
- Greatest Common Factor (GCF)
- Difference of Two Perfect Squares (DOTS)
- Trinomial (TRI)  
  - Case 1
- “AC” Method  
  - Case 2
- Quadratic Formula (QF)

**GCF:**

\[ ab + ac = a(b + c) \]

**DOTS:**

\[ x^2 - y^2 = (x + y)(x - y) \]

**TRI:**

\[ x^2 - x + 6 = (x + 2)(x - 3) \]

**AC (a≠1):**

\[ 2x^2 + 15x + 18 \]

\[ 2x^2 + 12x + 3x + 18 \]

\[ 2x(x + 6) \]

\[ 3(x + 6) \]

\[ (x + 6)(2x + 3) \]

**Sum of Two Squares:** \[ x^4 + 25 = (x + 5i)(x - 5i) \]

**QF:**

If all else fails to find the roots to a quadratic, use the Quadratic Formula:

\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]

**DIVIDING POLYNOMIALS**

**Division Algorithm:**

\[ \frac{Dividend}{Divisor} = Quotient + \frac{Remainder}{Divisor} \]

**Long Division of Polynomials:**

\[ (2x^2 + 7x + 6) \div (x + 2) \]

**Synthetic Division of Polynomials:**

\[ (x^3 + 6x^2 + 7x - 6) \div (x + 4) \]

**OTHER FORMS OF FACTORING**

- Factor by Grouping:
  
  \[ (x^2 + 2x)(x + 3) \]

- Factoring Perfect Cubes by SOAP:
  
  - S – “Same” as the sign in the middle of the original expression
  - O – “Opposite” sign
  - AP – “Always Positive”

\[ x^3 - 8 \]

\[ (x - 2)(x^2 + 2x + 4) \]

**THE REMAINDER THEOREM**

When the polynomial \( f(x) \) is divided by a binomial in the form of \( (x - a) \), the remainder equals \( f(a) \).

\[ \frac{f(a)}{x - 1} \]

\[ f(1) = 4(1)^2 + 2(1) - 5 \]

The remainder is 1!

**THE FACTOR THEOREM**

If \( f(a) = 0 \) for polynomial \( f(x) \), then a binomial in the form of \( (x - a) \) must be a factor of the polynomial.

\[ x^4 + 6x^3 + 7x^2 - 6x - 8 \]

\[ (x + 4) \]

\[ f(-4) = (-4)^4 + 6(-4)^3 + 7(-4)^2 - 6(-4) - 8 \]

\[ f(-4) = 256 + (-384) + 112 - (-24) - 8 \]

\[ f(-4) = 0 \]

The remainder is zero, therefore \( (x + 4) \) is a factor!
**QUADRATIC:** A *quadratic equation* is a polynomial equation with a degree of two (2).

### THE STANDARD FORM OF A QUADRATIC EQUATION

The standard form of a quadratic is in the form of

\[ ax^2 + bx + c = 0, \]

where \( a, b, \) and \( c \) are constants where \( a \neq 0 \).

### THE DISCRIMINANT

The discriminant is a part of the quadratic formula which allows mathematicians to anticipate the nature, or kinds of roots a particular quadratic equation will have.

\[ b^2 - 4ac \]

where \( a, b, \) and \( c \) are constants.

<table>
<thead>
<tr>
<th>The Value of the Discriminant</th>
<th>The Nature of the Roots</th>
<th>Number of X-Intercepts</th>
</tr>
</thead>
<tbody>
<tr>
<td>( b^2 - 4ac &gt; 0 ), and is a perfect square</td>
<td>Real, Rational, &amp; Unequal</td>
<td>2</td>
</tr>
<tr>
<td>( b^2 - 4ac &gt; 0 ), and is not a perfect square</td>
<td>Real, Irrational, &amp; Unequal</td>
<td>2</td>
</tr>
<tr>
<td>( b^2 - 4ac &lt; 0 )</td>
<td>Imaginary</td>
<td>0 (never touches the x-axis)</td>
</tr>
<tr>
<td>( b^2 - 4ac = 0 )</td>
<td>Real, Rational, &amp; Equal</td>
<td>1 (multiplicity of 2, called a <strong>bounce</strong>)</td>
</tr>
</tbody>
</table>

### THE SUM OF THE ROOTS OF A QUADRATIC

**Sum of the Roots:** \( r_1 + r_2 = \frac{-b}{a} \)

where \( a \) and \( b \) are constants from a quadratic equation in the form of \( ax^2 + bx + c = 0 \).

### THE PRODUCT OF THE ROOTS OF A QUADRATIC

**Product of the Roots:** \( r_1 \cdot r_2 = \frac{c}{a} \)

where \( a \) and \( c \) are constants from a quadratic equation in the form of \( ax^2 + bx + c = 0 \).

---

**The Parts of a Quadratic**

- **Axis of Symmetry**
- **Root Zero X-Intercept**
- **Focus**
- **Directrix**
- **Turning Point**

**Vertex Form - (vertex at \((h,k)\))**

\[ y = a(x-h)^2 + k \]

**Focus:** \( a = \frac{1}{4f} \)

*Remember, the vertex is in the middle of the focus & directrix*
**FUNCTION:** A function is a relation that consists of a set of ordered pairs in which each value of \( x \) is connected to a unique value of \( y \) based on the rule of the function. For each \( x \) value, there is one and only one corresponding value of \( y \). A function also passes the vertical line test.

**DOMAIN:** The largest set of elements available for the independent variable, the first member of the ordered pair \((x)\).

**Restrictions on Domain:**
1. **Fraction:** The denominator cannot be zero.
   Set the entire denominator equal to zero and solve.
   \[ f(x) = \frac{x - 4}{x + 3}; \quad x \neq -3 \]
2. **Radical:** The radicand cannot be negative.
   Set the radicand greater than or equal to zero and solve.
   \[ f(x) = \sqrt{x - 5}; \quad x \geq 5 \]
3. **Radical in the Denominator:** The radicand cannot be negative and the denominator cannot be zero.
   Set the radicand greater than zero and solve.
   \[ f(x) = \frac{1}{\sqrt{x} + 7}; \quad x > -7 \]

**RANGE:** The set of elements for the dependent variable, the second member of the ordered pair \((y)\).

**COMPOSITION FUNCTIONS:** One function is substituted into another in place of the variable. This can involve numeric substitutions or substitutions of an algebraic expression in the function in the place of the variable.

**Notation:** \( f(g(x)) \) or \( f \circ g(x) \)
Always read from right to left when using this notation.

**Example 1:** If \( f(x) = x + 9 \) and \( g(x) = 2x + 3 \), find \( f(g(3)) \)
\[ g(3) = 2(3) + 3 \Rightarrow 6 + 3 = 9 \]
\[ f(9) = (9) + 9 = 18 \]

**Example 2:** If \( f(x) = x + 5 \) and \( g(x) = 3x + 4 \), find \( g \circ f(x) \)
\[ f(x) = x + 5 \]
\[ g(x + 5) = 3(x + 5) + 4 \Rightarrow 3x + 15 + 4 = 3x + 19 \]

**One-to-One Function**
A one-to-one function must be a function, where when the ordered pairs are examined, the are no repeating \( x \) values or \( y \) values. One-to-one functions also pass both the horizontal and vertical line tests.

**Onto Function**
All \( x \) values and all \( y \) values are used.

**Inverse Functions:**
The inverse of a function is the reflection of the function over the line \( y = x \). Only a one-to-one function has an inverse function.

**Notation:**
\( f(x) \) is the function
\( f^{-1}(x) \) is the inverse

© NYS Mathematics Regents Preparation || Revised Spring 2017 || www.nysmathregentsprep.com
**END BEHAVIOR**

The *end behavior* of a graph is defined as what direction the function is heading at the ends of the graph. The end behavior can be determined by the following:

1. The degree of the function
2. The leading coefficient of the function

**NOTATION:**

\[ \text{As } x \to \pm \infty, \ f(x) \to \pm \infty \]

This notation is read as “As \(x\) approaches positive/negative infinity, \(y\) approaches positive/negative infinity.”

(*NOTE*: In Algebra 2, these are the only two notations you should know)

---

**MULTIPlicity**

*Multiplicity* is defined as how many times a particular number is a zero for a given polynomial. In other words, it’s the amount of times a root repeats itself given the features of the function.

**Odd Degree Polynomials**

**Positive Leading Coefficient**

\[ \text{As } x \to -\infty, \ f(x) \to -\infty \]

\[ \text{As } x \to \infty, \ f(x) \to \infty \]

**Negative Leading Coefficient**

\[ \text{As } x \to -\infty, \ f(x) \to \infty \]

\[ \text{As } x \to \infty, \ f(x) \to -\infty \]

**Even Degree Polynomials**

**Positive Leading Coefficient**

\[ \text{As } x \to -\infty, \ f(x) \to \infty \]

\[ \text{As } x \to \infty, \ f(x) \to \infty \]

**Negative Leading Coefficient**

\[ \text{As } x \to -\infty, \ f(x) \to -\infty \]

\[ \text{As } x \to \infty, \ f(x) \to -\infty \]
**COMPLEX NUMBERS**
The imaginary unit, \( i \), is the number whose square is negative one.

\[ \sqrt{-1} = i \quad \iff \quad i^2 = -1 \]

To solve for a value of \( i \), you can use your calculator or you can use the \( i \)-clock!

**Example: Solve for \( i^7 \)**

To solve, start at the top (\( i^0 \)) and count around the clock at each quarter interval, and stop when you reach \( i^7 \). The answer is \(-i\).

---

**RATIONAL EXPRESSIONS & EQUATIONS**
To add or subtract rational expressions, you need to find a **common denominator**!

\[
\frac{10}{2x^2} + \frac{5}{3x} = \frac{2}{3} \cdot \frac{10}{2x^2} + \frac{5}{3x} = \frac{30 + 10x}{6x^2}
\]

To multiply rational expressions, factor first, reduce, and then multiply through.

\[
\frac{6a}{3a + 15} \cdot \frac{4a + 20}{2a^2} = \frac{2}{3(a + 5)} \cdot \frac{2(a + 5)}{2a^2} = \frac{1}{a} = \frac{4}{a}
\]

To divide rational expressions, flip the second fraction, factor, reduce, and then multiply through.

\[
\frac{6x + 18}{4} \div \frac{x^2 + 3x}{5x^2} = \frac{3(x + 3)}{x(x + 3)} = \frac{15x}{2}
\]

**Complex Fractions:**
Remember, a fraction is division. They follow the same rules.

\[
\frac{1}{\frac{x + 5}{x^2 + 15}} = \frac{1}{5} \cdot \frac{x + 5}{3} = \frac{x + 5}{3}
\]

---

**PROPERTIES OF EXPONENTS & RADICALS**

\[
x^0 = 1
\]

\[
x^m \cdot x^n = x^{m+n}
\]

\[
x^{-m} = \frac{1}{x^m}
\]

\[
(x^n)^m = x^{nm}
\]

\[
\frac{x^m}{x^n} = x^{m-n}
\]

\[
(x \div y)^n = \frac{x^n}{y^n}
\]

---

**LOGARITHMS**

\[
B^e = N \quad \iff \quad \log_B N = e
\]

An exponent and a logarithm are inverses of each other!

**Properties of Logarithms**

\[
\log_b (m \cdot n) = \log_b m + \log_b n
\]

\[
\log_b \left(\frac{m}{n}\right) = \log_b m - \log_b n
\]

\[
\log_b m^r = r \log_b m
\]

\[
\log_b b = 1
\]

\[
\log_b 1 = 0
\]

**Properties of Natural Logarithms**

\[
\ln(ab) = \ln a + \ln b
\]

\[
\ln \left(\frac{a}{b}\right) = \ln a - \ln b
\]

\[
\ln a^b = b \ln a
\]

\[
\ln 1 = 0
\]

\[
\ln e = 1
\]
TRIGONOMETRY & TRIGONOMETRIC FUNCTIONS

SOH CAH TOA
\[ \sin \theta = \frac{\text{opposite}}{\text{hypotenuse}} \quad \cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}} \quad \tan \theta = \frac{\text{opposite}}{\text{adjacent}} \]

RADIANS
To change from degrees to radians, multiply by \frac{\pi}{180}.

DEGREES
To change from radians to degrees, multiply by \frac{180}{\pi}.

THE UNIT CIRCLE

ARC LENGTH OF A CIRCLE
\[ s = r \cdot \theta \]
where \( s \) is the length of the sector, \( r \) is the length of the radius, and \( \theta \) is an angle in radians.

SPECIAL RIGHT TRIANGLES

TRIGONOMETRIC FUNCTIONS

\[
\begin{align*}
\sin \theta &= \frac{\text{opposite}}{\text{hypotenuse}} \\
\cos \theta &= \frac{\text{adjacent}}{\text{hypotenuse}} \\
\tan \theta &= \frac{\text{opposite}}{\text{adjacent}} \\
\csc \theta &= \frac{1}{\sin \theta} \\
\sec \theta &= \frac{1}{\cos \theta} \\
\cot \theta &= \frac{1}{\tan \theta}
\end{align*}
\]

THE UNIT CIRCLE – EXACT VALUES

Remember these facts & the table below!

\[
\begin{array}{c|cc|cc|cc|c}
\theta & 0 & 90' & 180' & 270' & 360' \\
\sin \theta & 0 & 1 & 0 & -1 & 0 \\
\cos \theta & 1 & 0 & -1 & 0 & 1 \\
\tan \theta & 0 & \text{UNDEF} & 0 & \text{UNDEF} & 0 \\
\end{array}
\]

SPECIAL RIGHT TRIANGLES – EXACT VALUES

Remember the fractions below. The values can be checked with your calculator.

\[
\begin{align*}
\frac{\sqrt{2}}{2} &= 0.707 \\
\frac{\sqrt{3}}{2} &= 0.866 \\
\frac{1}{2} &= 0.5
\end{align*}
\]
TRIGONOMETRIC GRAPHS

THE TANGENT GRAPH

\[ y = A \tan(B(x - C)) + D \]

Amplitude (A): \( \frac{1}{2} |Maximum - Minimum| \)

Frequency (B): The number cycles the graph completes in \( 2\pi \) radians.

Horizontal Shift (C): The movement of a function left or right. The sign used in the equation is opposite the direction of the function.

Vertical Shift (D): The movement of a function up or down. The sign used in the equation is the same direction of the function.

Period: The distance to complete 1 full cycle. Formula: \( BP = 2\pi \)

THE PYTHAGOREAN IDENTITIES

\[ \sin^2 \theta + \cos^2 \theta = 1 \]

Square Notation

\[ \sin^2(\theta) = (\sin(\theta))^2 \]
\[ \cos^2(\theta) = (\cos(\theta))^2 \]
\[ \tan^2(\theta) = (\tan(\theta))^2 \]

THE QUADRANTS & TRIGONOMETRIC RELATIONSHIPS

QUADRANT I: All trigonometric functions are positive
QUADRANT II: Only sine is positive
QUADRANT III: Only tangent is positive
QUADRANT IV: Only cosine is positive
### SEQUENCES & SERIES

**SIGMA NOTATION:** Sigma Notation is used to write a series in a shorthand form. It is used to represent the sum of a number of terms having a common form. The diagram below shows the parts of a sigma notation (otherwise known as a summation).

Keywords: sum, total

- **Number of ending term in the series:** $\sum_{i=1}^{n}$
- **Expression to generate the sequence of terms to be added:** $2i$
- **Any variable representing integer values:** $i$
- **Symbol for SUM:** $\Sigma$

**Example:** Evaluate $\sum_{n=2}^{5} (3n - 2)$

\[
(3(2) - 2) + (3(3) - 2) + (3(4) - 2) + (3(5) - 2) = 4 + 7 + 10 + 13 = 34
\]

### FORMULAS

#### SUM OF FINITE SEQUENCES

**Arithmetic Series Formula:**

\[
S_n = \frac{n}{2} (a_1 + a_n)
\]

where $n$ is the number of terms in the sum, $a_1$ is the first term, and $a_n$ is the $n^{th}$ term in the sum.

**Geometric Series Formula:**

\[
S_n = \frac{a_1 (1 - r^n)}{1 - r}
\]

where $r$ is the common ratio and $r \neq 1$, $n$ is the number of terms in the sum, $a_1$ is the first term.

### REMEMBER!

- **Common Difference ($d$):** $a_2 - a_1$
- **Common Ratio ($r$):** $\frac{a_2}{a_1}$

<table>
<thead>
<tr>
<th>Arithmetic Sequences</th>
<th>Geometric Sequences</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_n = a_1 + (n - 1)d$</td>
<td>$a_n = a_1 \cdot (r)^{n-1}$</td>
</tr>
</tbody>
</table>

where "$a_1$" is the first term of the sequence, "$n$" is the desired term, and "$d$" is the common difference.

where "$a_1$" is the first term of the sequence, "$n$" is the desired term, and "$r$" is the common ratio.

<table>
<thead>
<tr>
<th>Explicit Formula</th>
<th>Recursive Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_n = a_1 + (n - 1)d$</td>
<td>$a_n = a_{n-1} + d$</td>
</tr>
<tr>
<td>where &quot;$a_1$&quot; is the first term of the sequence, &quot;$n$&quot; is the desired term, and &quot;$d$&quot; is the common difference.</td>
<td>where &quot;$a_1$&quot; is the first term of the sequence, &quot;$n$&quot; is the desired term, and &quot;$r$&quot; is the common ratio.</td>
</tr>
</tbody>
</table>

**DEFINITIONS**

**Sequence:** a list of terms or elements in order. The terms are identified using positive integers as subscripts of $a$: $a_1, a_2, a_3, \ldots a_n$. The terms in a sequence can form a pattern or they can be random.

**Series:** the sum of all the terms of a sequence.

**Explicit Formula:** If specific terms are not given, a formula, sometimes called an explicit formula, is given. It can be used by substituting the number of the term desired into the formula for "$n$".

**Recursive Formula:** In a recursive formula, the first term in a sequence is given and subsequent terms are defined by the term before it. If $a_n$ is the term we are looking for, $a_{n-1}$, which is the term before $a_n$, must be used.
STATISTICS & PROBABILITY

TYPES OF STATISTICAL STUDIES
Survey: used to gather large quantities of facts or opinions. Surveys are usually asked in the form of a question. For example, “Do you like Algebra, Geometry, or neither?” would be a survey question.
Observational Study: the observer does not have any interaction with the subjects and just examines the results of an activity. For example, the location as to where the Sun rises and sets on each day throughout the year.
Controlled Experiment: two groups are studied while an experiment is performed with one of them but not the other. For example, testing if orange juice has an effect in preventing the “common cold” with a group of 100 people, where 50 people will drink orange juice and the other 50 will not drink the juice. The statistician will then analyze the data of the control group and the experimental group.

SET NOTATION IN PROBABILITY
Universal Set
Intersection Set of A AND B
Union Set of A OR B

THE NORMAL DISTRIBUTION CURVE
Standard Deviation: Measures how far the data is spread from the mean. Symbol: σx
95% is 2σx
Whenever you are asked if things are: NORMAL, FAIR, or EXPECTED, check if the given number is within 2σx

INDEPENDENT & DEPENDENT EVENTS OF PROBABILITY
Independent Event: Two events are independent if one happening (or not happening) has nothing to do whether or not the other happens (or doesn’t happen).
Dependent Event: Two events are dependent if the outcome or occurrence of the first affects the outcome or occurrence of the second so that the probability is changed.

CONFIDENCE INTERVALS
A Confidence interval is a range or interval of values used to estimate the true value of a population parameter. The formula to calculate the confidence interval is given by:
where ≠ is a known value, μ is the mean, and σ changes value depending on the confidence level.

INDEPENDENT & DEPENDENT EVENTS OF PROBABILITY
1) If A and B are mutually exclusive events, 
P(A or B) = P(A) + P(B)
2) If events A and B are NOT mutually exclusive,
P(A or B) = P(A) + P(B) – P(A and B)

CONDITIONAL PROBABILITY
The conditional probability of an event B, in relation to event A, is the probability that event B will occur given the knowledge that an event A has already occurred.
P(B|A) = \frac{P(A\cap B)}{P(B)} = \frac{\text{Both}}{\text{Back}}

NOTATION: P(B|A)
Read as “the probability of B given A”