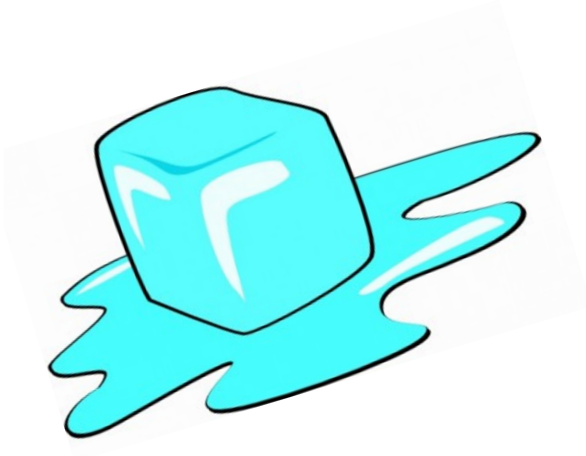


ALGEBRA II (COMMON CORE)

FACTS YOU MUST KNOW COLD FOR THE REGENTS EXAM

ALGEBRA & FUNCTIONS

FACTORING

The Order of Factoring:

Greatest Common Factor (GCF)
↓
Difference of Two Perfect Squares (DOTS)
↓
Trinomial (TRI) (Case 1)
↓
"AC" Method / (Case 2)
↓
Quadratic Formula (QF)

GCF:

$$ab + ac = a(b + c)$$

DOTS:

$$x^2 - y^2 = (x + y)(x - y)$$

TRI:

$$x^2 - x + 6 = (x + 2)(x - 3)$$

AC (a≠1):

$$2x^2 + 15x + 18$$

$ac = 36$
$12 \quad 3$

$2x^2 + 12x \mid + 3x + 18$

$2x(x + 6) \mid 3(x + 6)$

$(x + 6)(2x + 3)$

Sum of Two Squares: $x^2 + 25 = (x + 5i)(x - 5i)$

QF:

If all else fails to find the roots to a quadratic, use the Quadratic Formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

DIVIDING POLYNOMIALS

Division Algorithm: $\dfrac{Dividend}{Divisor} = Quotient + \dfrac{Remainder}{Divisor}$

Long Division of Polynomials:

$$(2x^2 + 7x + 6) \div (x + 2)$$

multiply

2x was needed to create the first term of $2x^2$

$$x + 2 \overline{)\,2x^2 + 7x + 6\,} \quad \text{quotient: } 2x + 3$$

$-(2x^2 + 4x)$ ←(subtract)

$3x + 6$

$-(3x + 6)$ ←(subtract)

0 remainder

$\boxed{2x + 3}$

Synthetic Division of Polynomials:

$$(x^3 + 6x^2 + 7x - 6) \div (x + 4)$$

Can ONLY be used with normal x

$x + 4 = 0$
$\;\;\;-4 \;\; -4$
$\boxed{x = -4}$

-4	1	6	7	-6
		-4	-8	4
	1	2	-1	$\boxed{-2}$

multiply · Add · Add · Add · Remainder

$\boxed{x^2 + 2x - 1 + \frac{-2}{x+4}}$

OTHER FORMS OF FACTORING

Factor by Grouping:

$x^3 + 2x^2 \mid - 3x - 6$

$x^2(x + 2) \mid - 3(x + 2)$

$\boxed{(x^2 - 3)(x + 2)}$

Factoring Perfect Cubes by SOAP:

S – "Same" as the sign in the middle of the original expression"

O – "Opposite" sign

AP – "Always Positive"

$$x^3 - 8$$
$$(x)^3 - (2)^3$$
$$\boxed{(x - 2)(x^2 + 2x + 4)}$$

Perfect Cube Factor SOAP Factor

THE REMAINDER THEOREM

When the polynomial $f(x)$ is divided by a binomial in the form of $(x - a)$, the remainder equals $f(a)$.

$$\frac{4x^2 + 2x - 5}{(x - 1)}$$

$$f(1) = 4(1)^2 + 2(1) - 5 \Rightarrow \boxed{1}$$

The remainder is 1!

THE FACTOR THEOREM

If $f(a) = 0$ for polynomial $f(x)$, then a binomial in the form of $(x - a)$ must be a factor of the polynomial.

$$\frac{x^4 + 6x^3 + 7x^2 - 6x - 8}{(x + 4)}$$

$$f(-4) = (-4)^4 + 6(-4)^3 + 7(-4)^2 - 6(-4) - 8$$
$$f(-4) = 256 + (-384) + 112 - (-24) - 8$$
$$f(-4) = 0$$

The remainder is zero, therefore $(x + 4)$ is a factor!

QUADRATIC: A *quadratic equation* is a polynomial equation with a degree of two (2).

THE STANDARD FORM OF A QUADRTATIC EQUATION

The standard form of a quadratic is in the form of

$$ax^2 + bx + c = 0,$$

where a, b, and c are constants where a ≠ 0.

THE DISCRIMINANT

The discriminant is a part of the quadratic formula which allows mathematicians to anticipate the nature, or kinds of roots a particular quadratic equation will have.

$$b^2 - 4ac$$

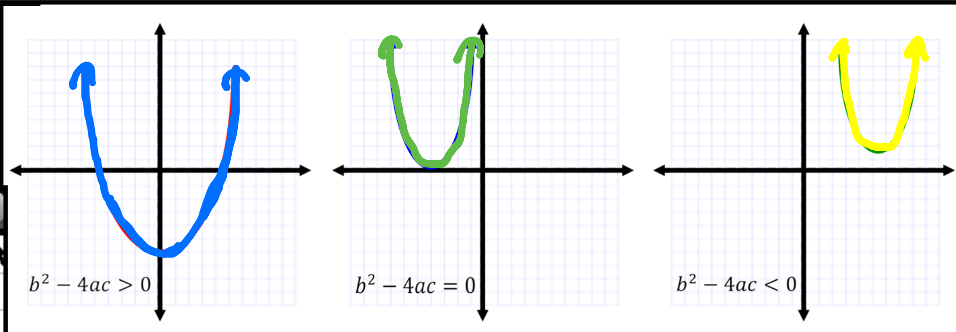
where a, b, and c are constants

The Value of the Discriminant	The Nature of the Roots	Number of X-Intercepts
$b^2 - 4ac > 0$, and is a perfect square	Real, Rational, & Unequal	2
$b^2 - 4ac > 0$, and is *not* a perfect square	Real, Irrational, & Unequal	2
$b^2 - 4ac < 0$	Imaginary	0 (never touches the x-axis)
$b^2 - 4ac = 0$	Real, Rational, & Equal	1 (multiplicity of 2, called a *bounce*)

THE SUM OF THE ROOTS OF A QUADRATIC

Sum of the Roots: $r_1 + r_2 = \frac{-b}{a}$

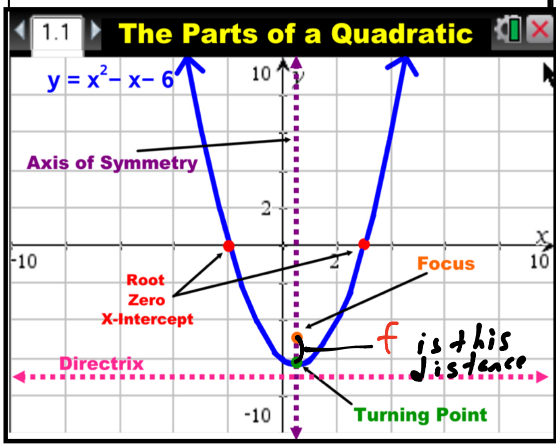
where a and b are constants from a quadratic equation in the form of $ax^2 + bx + c = 0$.



THE PRODUCT OF THE ROOTS OF A QUADRATIC

Product of the Roots: $r_1 \cdot r_2 = \frac{c}{a}$

where a and c are constants from a quadratic equation in the form of $ax^2 + bx + c = 0$.



Vertex Form. (vertex at (h,k))

$y = a(x-h)^2 + k$

Focus: $a = \frac{1}{4f}$

* Remember, the vertex is in the middle of the focus & directrix

FUNCTION: A function is a relation that consists of a set of ordered pairs in which each value of x is connected to a unique value of y based on the rule of the function. For each x value, there is one and only one corresponding value of y. A function also passes the vertical line test.

DOMAIN: The largest set of elements available for the independent variable, the first member of the ordered pair (x).

RESTRICTIONS ON DOMAIN:

1. **Fraction:** The denominator cannot be zero. Set the entire denominator equal to zero and solve.

$$f(x)=\frac{x-4}{x+3};\ \ x\neq -3$$

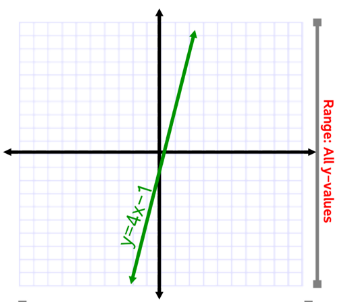
2. **Radical:** The radicand cannot be negative. Set the radicand greater than or equal to zero and solve.

$$f(x)=\sqrt{x-5};\ \ x\geq 5$$

3. **Radical in the Denominator:** The radical cannot be negative *and* the denominator cannot be zero. Set the radicand greater than zero and solve.

$$f(x)=\frac{1}{\sqrt{x+7}};\ \ x>-7$$

RANGE: The set of elements for the dependent variable, the second member of the ordered pair (y).



COMPOSITION FUNCTIONS: One function is substituted into another in place of the variable. This can involve numeric substitutions or substitutions of an algebraic expression in the function in the place of the variable.

NOTATION: $f(g(x))$ or $f\circ g(x)$

Always read from right to left when using this notation.

Example 1: If $f(x)=x+9$ and $g(x)=2x+3$, find $f(g(3))$

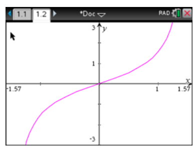
$$g(3)=2(3)+3\Longrightarrow 6+3=\mathbf{9}$$
$$f(9)=(9)+9=\boxed{\mathbf{18}}$$

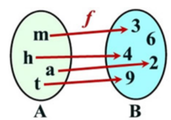
Example 2: If $f(x)=x+5$ and $g(x)=3x+4$, find $g\circ f(x)$

$$f(x)=x+5$$
$$g(x+5)=3(x+5)+4\Rightarrow 3x+15+4=\boxed{3x+19}$$

ONE-TO-ONE FUNCTION

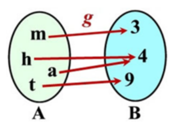
A one-to-one function must be a function, where when the ordered pairs are examined, the are no repeating x values or y values. One-to-one functions also pass *both* the horizontal and vertical line tests.





ONTO FUNCTION

All x values and all y values are used.



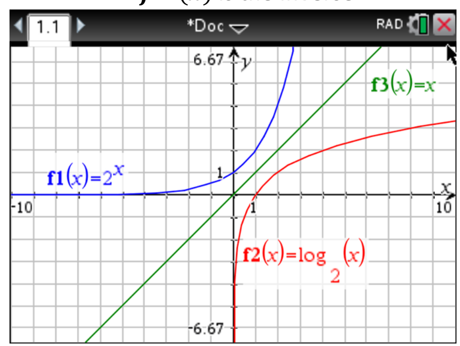
INVERSE FUNCTIONS:

The inverse of a function is the reflection of the function over the line $y=x$. Only a one-to-one function has an inverse function.

NOTATION:

$f(x)$ is the function

$f^{-1}(x)$ is the inverse



END BEHAVIOR

The *end behavior* of a graph is defined as what direction the function is heading at the ends of the graph. The end behavior can be determined by the following:

1. The degree of the function
2. The leading coefficient of the function

NOTATION:

As $x \to \pm\infty$, $f(x) \to \pm\infty$

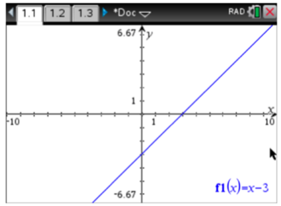
This notation is read as. "As x approaches positive/negative infinity, y approaches positive/negative infinity."

(*NOTE*: In Algebra 2, these are the only two notations you should know)

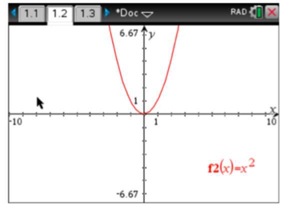
MULTIPLICITY

Multiplicity **is defined as how many times a particular number is a zero for a given polynomial. In other words, it's the amount of times a root repeats itself given the features of the function.**

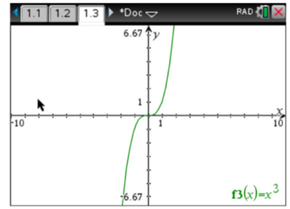
Multiplicity of 1 "Cut Thru"



Multiplicity of 2 "Bounce"



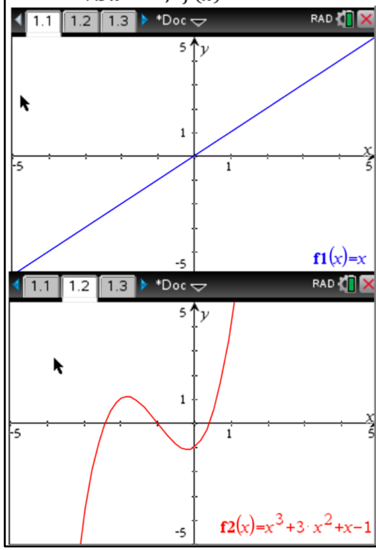
Multiplicity of 3 "Snake Thru"



Odd Degree Polynomials

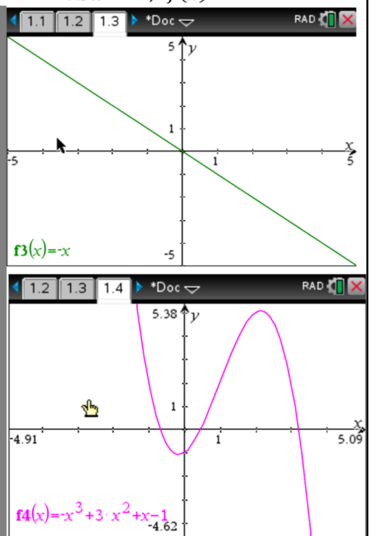
Positive Leading Coefficient

As $x \to -\infty$, $f(x) \to -\infty$
As $x \to \infty$, $f(x) \to \infty$



Negative Leading Coefficient

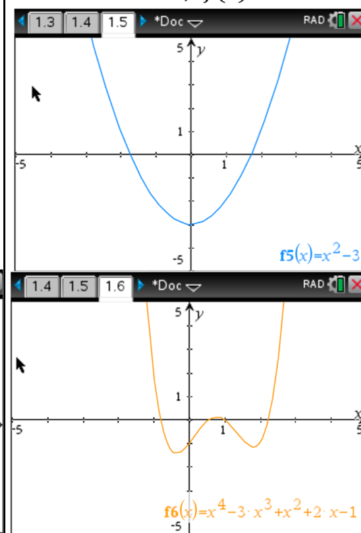
As $x \to -\infty$, $f(x) \to \infty$
As $x \to \infty$, $f(x) \to -\infty$



Even Degree Polynomials

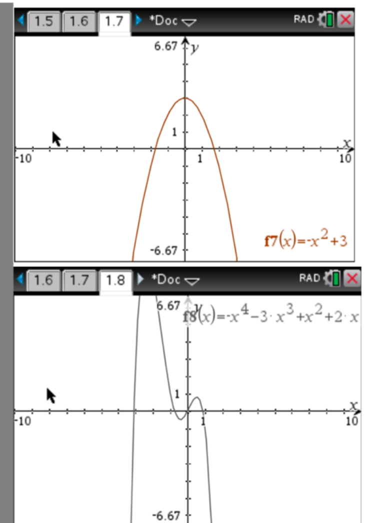
Positive Leading Coefficient

As $x \to -\infty$, $f(x) \to \infty$
As $x \to \infty$, $f(x) \to \infty$



Negative Leading Coefficient

As $x \to -\infty$, $f(x) \to -\infty$
As $x \to \infty$, $f(x) \to -\infty$

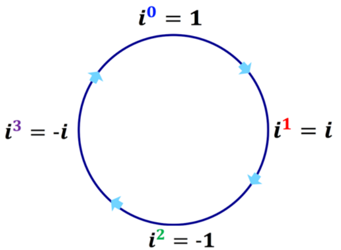


COMPLEX NUMBERS

The imaginary unit, i, is the number whose square is negative one.

$$\sqrt{-1} = i \qquad \Leftrightarrow \qquad i^2 = -1$$

The i-Clock



To solve for a value of i, you can use your calculator or you can use the i –clock!

Example: Solve for i^7

To solve, start at the top (i^0) and count around the clock at each quarter interval, and stop when you reach i^7. The answer is $-\boldsymbol{i}$.

RATIONAL EXPRESSIONS & EQUATIONS

To add or subtract rational expressions, you need to find a *common denominator!*

$$\frac{10}{2x^2} + \frac{5}{3x} \Rightarrow \frac{3}{3} \cdot \frac{10}{2x^2} + \frac{5}{3x} \cdot \frac{2x}{2x} \Rightarrow \frac{30}{6x^2} + \frac{10x}{6x^2} = \boxed{\frac{30+10x}{6x^2}}$$

To multiply rational expressions, factor first, reduce, and then multiply through.

$$\frac{6a}{3a+15} \cdot \frac{4a+20}{2a^2} \Rightarrow \frac{\overset{2}{\cancel{6a}}}{\underset{1}{\cancel{3}}\cancel{(a+5)}} \cdot \frac{\overset{2}{\cancel{4}}\cancel{(a+5)}}{\cancel{2a^2}_{a}} \Rightarrow \frac{2}{1} \cdot \frac{2}{a} = \boxed{\frac{4}{a}}$$

To divide rational expressions, flip the second fraction, factor, reduce, and then multiply through.

$$\frac{6x+18}{4} \div \frac{x^2+3x}{5x^2} \Rightarrow \frac{6x+18}{4} \cdot \frac{5x^2}{x^2+3x} \Rightarrow \frac{\overset{3}{\cancel{6}}\cancel{(x+3)}}{2\cancel{4}} \cdot \frac{5\overset{x}{\cancel{x^2}}}{\cancel{x}\cancel{(x+3)}} = \boxed{\frac{15x}{2}}$$

Complex Fractions:

Remember, a fraction is division. They follow the same rules.

$$\frac{\frac{1}{x+5}}{\frac{3}{x^2-25}} = \frac{1}{\cancel{x+5}} \cdot \frac{\overset{(x+5)(x-5)}{\cancel{x^2-25}}}{3} = \boxed{\frac{x-5}{3}}$$

PROPERTIES OF EXPONENTS & RADICALS

$x^0 = 1$

$x^m \cdot x^n = x^{m+n}$

$x^{-m} = \dfrac{1}{x^m}$

$(x^n)^m = x^{n \cdot m}$

$\dfrac{x^m}{x^n} = x^{m-n}$

$\left(\dfrac{x}{y}\right)^n = \dfrac{x^n}{y^n}$

$(xy)^n = x^n \cdot y^n$

$x^{\frac{p}{r}} = \sqrt[r]{x^p}$

$\sqrt[a]{x} = x^{\frac{1}{a}}$

$\sqrt[n]{a^n} = a$

$\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$

$\sqrt[n]{\dfrac{a}{b}} = \dfrac{\sqrt[n]{a}}{\sqrt[n]{b}}$

LOGARITHMS

$$B^e = N \qquad \Leftrightarrow \qquad \log_B N = e$$

Exponential Form — Logarithmic Form

An exponent and a logarithm are *inverses* of each other!

Properties of Logarithms

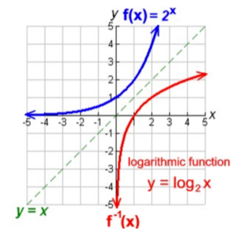
$\log_b(m \cdot n) = \log_b m + \log_b n$

$\log_b\left(\frac{m}{n}\right) = \log_b m - \log_b n$

$\log_b m^r = r \log_b m$

$\log_b b = 1$

$\log_b 1 = 0$



Properties of Natural Logarithms

$\ln(ab) = \ln a + \ln b$

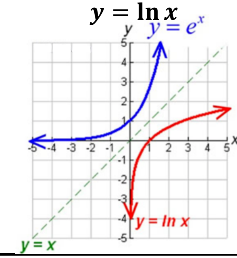
$\ln\left(\frac{a}{b}\right) = \ln a - \ln b$

$\ln a^b = b \ln a$

$\ln 1 = 0$

$\ln e = 1$

The inverse of $y = e^x$ is $y = \ln x$



TRIGONOMETRY & TRIGONOMETRIC FUNCTIONS

TRIGONOMETRIC FUNCTIONS

SOH CAH TOA

$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$ $\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$ $\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$

RECIPROCAL FUNCTIONS

Cosecant	Secant	Cotangent
$\csc\theta = \frac{1}{\sin\theta}$	$\sec\theta = \frac{1}{\cos\theta}$	$\cot\theta = \frac{1}{\tan\theta}$

RADIANS

To change from *degrees* to *radians*, multiply by $\frac{\pi}{180}$.

DEGREES

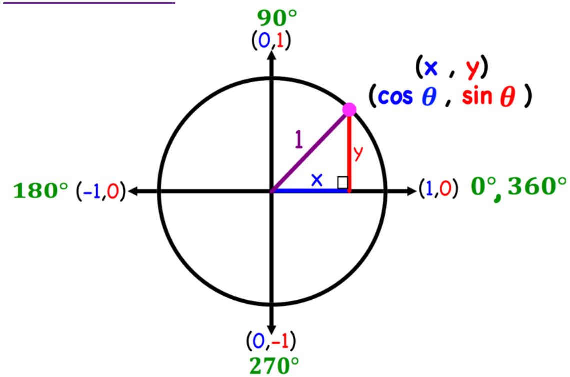
To change from *radians* to *degrees*, multiply by $\frac{180}{\pi}$.

ARC LENGTH OF A CIRCLE

$$s = r \cdot \theta$$

where s is the length of the sector, r is the length of the radius, and θ is an angle in radians

THE UNIT CIRCLE



THE UNIT CIRCLE – EXACT VALUES

Remember these facts & the table below!

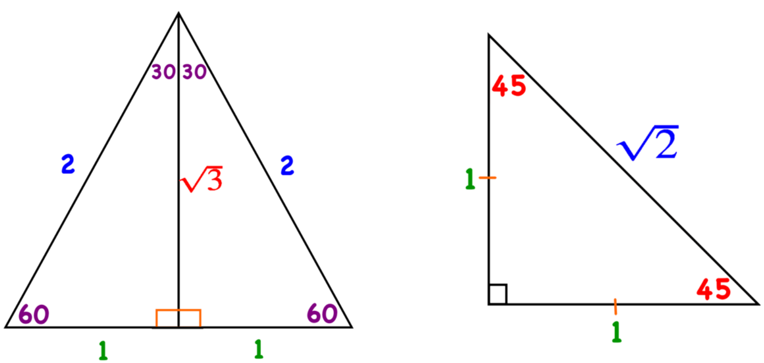
$\cos\theta = x$

$\sin\theta = y$

$\tan\theta = \frac{y}{x} = \frac{\sin\theta}{\cos\theta}$

θ	0°	90°	180°	270°	360°
sin θ	0	1	0	-1	0
cos θ	1	0	-1	0	1
tan θ	0	UNDEF	0	UNDEF	0

SPECIAL RIGHT TRIANGLES

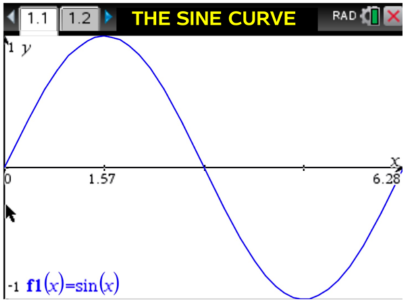


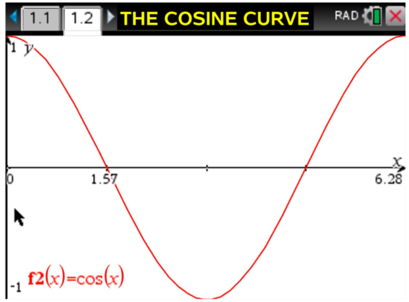
SPECIAL RIGHT TRIANGLES – EXACT VALUES

Remember the fractions below. The values can be checked with your calculator.

$\frac{\sqrt{2}}{2} = 0.707$ $\frac{\sqrt{3}}{2} = 0.866$ $\frac{1}{2} = 0.5$

TRIGONOMETRIC GRAPHS





STANDARD FORMS OF TRIGONOMETRIC GRAPHS

$$y = A\sin(B(x - C)) + D \qquad y = A\cos(B(x - C)) + D$$

Amplitude (A): $\frac{1}{2}|Maximum - Minimum|$

Frequency (B): The number cycles the graph completes in 2π radians.

Horizontal Shift (C): The movement of a function left or right. The sign used in the equation is opposite the direction of the function.

Vertical Shift (D): The movement of a function up or down. The sign used in the equation is the same direction of the function.

Period: The distance to complete 1 full cycle. Formula: $BP = 2\pi$

THE PYTHAGOREAN IDENTITIES

$$\sin^2\theta + \cos^2\theta = 1$$

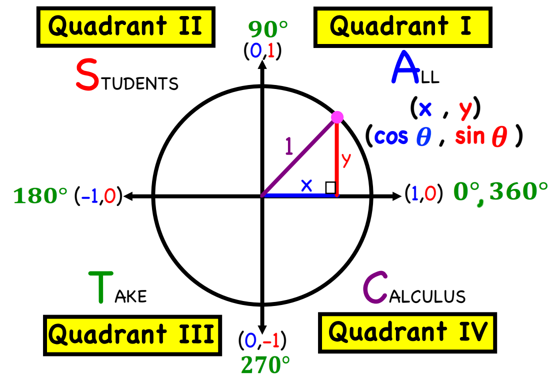
Square Notation

$\sin^2(\theta) = (\sin(\theta))^2$

$\cos^2(\theta) = (\cos(\theta))^2$

$\tan^2(\theta) = (\tan(\theta))^2$

THE QUADRANTS & TRIGONOMETRIC RELATIONSHIPS



QUADRANT I: All trigonometric functions are positive

QUADRANT II: Only sine is positive

QUADRANT III: Only tangent is positive

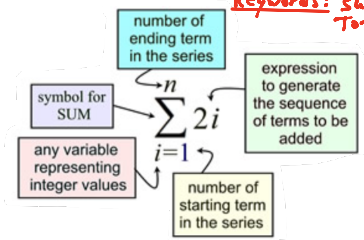
QUADRANT IV: Only cosine is positive

SEQUENCES & SERIES

SIGMA NOTATION:

Sigma Notation is used to write a series in a shorthand form. It is used to represent the *sum* of a number of terms having a common form. The diagram below shows the parts of a sigma notation (otherwise known as a *summation*).

Keywords: sum Total



Example: Evaluate $\sum_{n=2}^{5}(3n-2)$

$$(3(\mathbf{2})-2)+(3(\mathbf{3})-2)+(3(\mathbf{4})-2)+(3(\mathbf{5})-2)$$

$$(4)+(7)+(10)+(13)=\boxed{\mathbf{34}}$$

SUM OF FINITE SEQUENCES

Arithmetic Series Formula:

$$S_n=\frac{n(a_1+a_n)}{2}$$

where n is the number of terms in the sum, a_1 is the first term, and a_n is the nth term in the sum

Geometric Series Formula:

$$S_n=\frac{a_1(1-r^n)}{1-r}$$

where r is the common ratio and $r \neq 1$, n is the number of terms in the sum, a_1 is the first term.

DEFINITIONS

Sequence: a list of terms or elements in order. The terms are identified using positive integers as subscripts of a: $a_1, a_2, a_3, \ldots a_n$. The terms in a sequence can form a pattern or they can be random.

Series: the sum of all the terms of a sequence.

Explicit Formula: If specific terms are not given, a formula, sometimes called an explicit formula, is given. It can be used by substituting the number of the term desired into the formula for "n".

Recursive Formula: In a recursive formula, the first term in a sequence is given and subsequent terms are defined by the term before it. If a_n is the term we are looking for, a_{n-1}, which is the term *before* a_n, must be used.

FORMULAS

REMEMBER!

Common Difference (d): $a_2 - a_1$ **Common Ratio (r):** $\frac{a_2}{a_1}$

	Arithmetic Sequences	Geometric Sequences
Explicit Formula	$a_n = a_1 + (n-1)(d)$ where "a_1" is the first term of the sequence, "n" is the desired term, and "d" is the common difference.	$a_n = a_1 \cdot (r)^{n-1}$ where "a_1" is the first term of the sequence, "n" is the desired term, and "r" is the common ratio.
Recursive Formula	$a_1 = ?$ $a_n = a_{n-1} + d$ where "a_1" is the first term of the sequence, "n" is the desired term, and "d" is the common difference.	$a_1 = ?$ $a_n = a_{n-1} \cdot r$ where "a_1" is the first term of the sequence, "n" is the desired term, and "r" is the common ratio.

STATISTICS & PROBABILITY

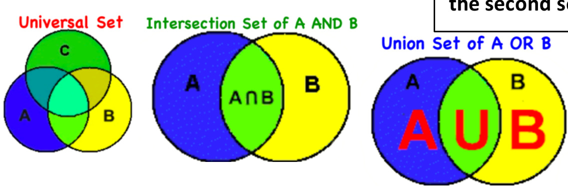
TYPES OF STATISTICAL STUDIES

Survey: used to gather large quantities of facts or opinions. Surveys are usually asked in the form of a question. For example, "Do you like Algebra, Geometry, or neither?" would be a survey question.

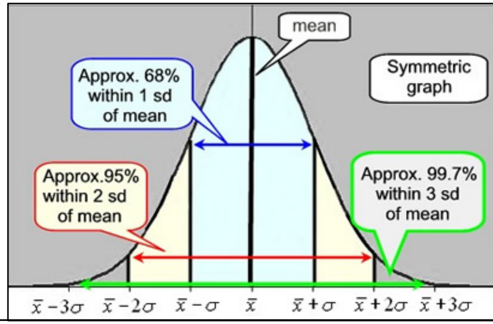
Observational Study: the observer does not have any interaction with the subjects and just examines the results of an activity. For example, the location as to where the Sun rises and sets on each day throughout the year.

Controlled Experiment: two groups are studied while an experiment is performed with one of them but not the other. For example, testing if orange juice has an effect in preventing the "common cold" with a group of 100 people, where 50 people will drink orange juice and the other 50 will not drink the juice. The statistician will then analyze the data of the control group and the experimental group.

SET NOTATION IN PROBABILITY



THE NORMAL DISTRIBUTION CURVE



Standard Deviation: Measures how far the data is spread from the mean. Symbol: σx

*95% is $2\sigma x$

Whenever you are asked if things are: NORMAL, FAIR, or EXPECTED

Check if the given number is within $2\sigma x$

INDEPENDENT & DEPENDENT EVENTS OF PROBABILITY

Independent Event: Two events are independent if one happening (or not happening) has nothing to do whether or not the other happens (or doesn't happen).

Dependent Event: Two events are dependent if the outcome or occurrence of the first affects the outcome or occurrence of the second so that the probability is changed.

Check for Dependence:

If $P(A) \cdot P(B) = P(A \cap B)$ - independent

- different means dependent

Area or % of data in a given interval

$\boxed{2ND}$ $\boxed{VARS}$ $\boxed{2}$

(lower, upper, mean standard deviation)

If no upper is given: use 999999

If no lower is given: use -999999

Margin of Error

Is $\pm 2\sigma x$

$\boxed{STAT}$ $\boxed{\triangleright}$ TEST $\boxed{A}$

for x: total * percent given

for n: use total

ALWAYS 95%

Use back number - $\hat{p}$

MUTUALLY EXCLUSIVE EVENTS IN PROBABILITY

1) If A and B are mutually exclusive events, $P(A\ or\ B) = P(A) + P(B)$
2) If events A and B are NOT mutually exclusive,

$P(A\ or\ B) = P(A) + P(B) - P(A\ and\ B)$

CONDITIONAL PROBABILITY

The conditional probability of an event B, in relation to event A, is the probability that event B will occur given the knowledge that an event A has already occurred. $P(B|A) = \frac{P(A \cap B)}{P(B)} = \frac{\text{Both}}{\text{Back}}$

NOTATION: $P(B|A)$

Read as "the probability of B given A"