In math, people often invent ways to extend concepts to areas that might not make sense at first. Pretty much everyone can understand what 2^3 means, because they understand that it represents multiplying by the number 2 a total of 3 times. Yet, what does 2^0 or 2^{-4} mean? Does it make sense to talk about multiplying by a number a negative amount of times? Lets explore.

***Exercise #1*:** We can think of powers of 2 as representing multiplication of the number 1 repeatedly.

(a) Fill in the pattern for powers that are not negative. What does this lead you to fill in for 2^0?

$2^4 = 1 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 16$

$2^3 = 1 \cdot 2 \cdot 2 \cdot 2 = 8$

$2^2 =$

$2^1 =$

$2^0 =$

(b) If positive exponents indicated multiplying the number 1 by 2 repeatedly, then negative exponents should indicate _____.

$2^{-1} = \frac{1}{2^1} = \frac{1}{2}$

$2^{-2} = \frac{1}{2^2} = \frac{1}{4}$

$2^{-3} =$

$2^{-4} =$

We want the pattern of positive, integer powers to extend to zero exponents and negative, integer exponents. We can now define zero and negative exponents as follows.

Zero & Negative Exponents	
1.) Zero Exponents: $b^0 =$ _____ as long as $b \neq 0$	2.) Negative Exponents: $b^{-n} =$ _____

***Exercise #2*:** Which of the following is **not** equivalent to 5^{-2}?

[1] $\frac{1}{5^2}$ [3] $\frac{1}{25}$

[2] $\frac{1}{10}$ [4] 0.04

Exercise #3*:** If $f(x) = 3x^{-2} + 2x^{0}$, then which of the following is the value of $f(2)$? Show the work that leads to your answer. Remember exponents ***always come before multiplication.

[1] $2\frac{3}{4}$ [3] $1\frac{1}{12}$

[2] $1\frac{3}{4}$ [4] $2\frac{1}{2}$

***Exercise #4*:** Using all your known exponent rules, simplify each of the following completely.

(a) $\frac{26x^5}{6x^2}$ (b) $\frac{3x^5}{9x^8}$ (c) $\frac{2x^4}{6x^4}$ (d) $\frac{16x^8}{12x^{15}}$

So, we now see that the subtraction rule for exponents is consistent with negative and zero exponents. For now, we just want to be comfortable that negative exponents indicate division and positive exponents indicate multiplication.

***Exercise #5*:** Consider the exponential function $f(x) = 16(2)^x$. Find each of the following without the use of your calculator.

(a) $f(0)$ (b) $f(2)$ (c) $f(-2)$

Homework

1.) Rewrite each of the following as equivalent expressions without the use of negative or zero exponents. Remember your order of operations.

(a) 5^{-3} (b) 6^{0} (c) 2^{-5}

(d) $4x^{0}$ (e) $(4x)^{0}$ (f) $x^{-2}y^{4}$

____ 2.) Which of the following is not equivalent to 2^{-3}?

[1] $\frac{1}{2^3}$ [3] 0.125

[2] -6 [4] $\frac{1}{8}$

____ 3.) If $f(x) = 12(2)^{x}$, then which of the following represents the value of $f(-2)$?

[1] -48 [3] 3

[2] 6 [4] -4

____ 4.) If the expression $8(x+11)^{0} - 2x^{0} + 6x$ is evaluated when $x = -1$, the result would be:

[1] 1 [3] 7

[2] 0 [4] 4

____ 5.) The numerical expression $\frac{(5x^3)^2}{(5x^2)^4}$ is equivalent to:

[1] $\frac{1}{25}$ [3] 10

[2] 25 [4] $-\frac{1}{10}$

6.) Write each of the following in the form ax^{n}, where n is either a positive or negative integer.

(a) $\frac{x^3}{x^8}$ (b) $\frac{6x}{2x^8}$ (c) $\frac{28x^6}{21x^2}$

7.) The number of people, n, ho know a rumor can be modeled using the equation $n(d) = 20(2)^d$, where d is the number of days *since* Monday.

(a) Explain why $n(0) = 20$. What does this represent in terms of the situation modeled?

(b) What is the value of $n(-2)$? What does this represent in terms of the situation modeled?

____ 8.) Consider the function $f(x) = 18(3)^{-x}$. When the value of x is increased by 1, the output is:

[1] multiplied by 3
[2] divided by 3
[3] multiplied by -3
[4] divided by -3

Review Section:

____ 9.) The inequality $7 - \frac{2}{3}x < x - 8$ is equivalent to

(1) $x > 9$
(2) $x > -\frac{3}{5}$
(3) $x < 9$
(4) $x < -\frac{3}{5}$

____ 10.) Two functions, $y = |x - 3|$ and $3x + 3y = 27$, are graphed on the same set of axes. Which statement is true about the solution to the system of equations?

(1) (3,0) is the solution to the system because it satisfies the equation $y = |x - 3|$.
(2) (9,0) is the solution to the system because it satisfies the equation $3x + 3y = 27$.
(3) (6,3) is the solution to the system because it satisfies both equations.
(4) (3,0), (9,0), and (6,3) are the solutions to the system of equations because they all satisfy at least one of the equations.

Homework Answers

1.) a) $\frac{1}{125}$ b) 1 c) $\frac{1}{32}$

d) 4 e)1 f) $\frac{y^4}{x^2}$

2.) 2

3.) 3

4.) 2

5.) 1

6.) a) $1x^{-5}$ b) $3x^{-7}$ c) $\frac{4}{3}x^4$

7.) a) $n(0) =$ 20 represents the fact that 20 people knew the rumor on Monday.
b) $n(-2) =$ 5 represents the fact that 5 people knew the rumor two days before Monday (Saturday).

8.) 2

9.) 1

10.) 3

So far we have concentrated on linear functions which are characterized by having a constant rate of change. We have also worked with quadratics ($f(x) = x^2$) where the rate of change is not constant. In this lesson we will more formally introduce the *concept of an exponential function.*

***Exercise #1*:** Consider the exponential function $f(x) = 8(2)^x$. Answer the following.

(a) Evaluate each of the following and indicate what point must lie on the graph of $f(x)$ based on each:

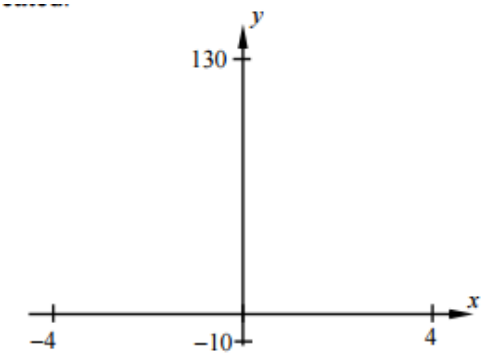
(i) $f(2)$ (ii) $f(0)$ (iii) $f(-1)$

(b) Calculate the average rate of change of f over the interval $-1 \leq x \leq 0$.

(c) Calculate the average rate of change over the interval $0 \leq x \leq 2$.

(d) What does comparing answers from (b) and (c) tell you about this function? Explain.

(e) Using your calculator, draw a sketch of this function on the axes below using the window indicated.



Exponential functions are all about ***multiplication***. The basic form of an exponential function is given below.

EXPONENTIAL FUNCTIONS

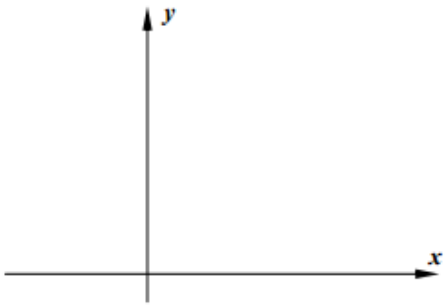
A general exponential function has the form: $y = a(b)^x$, where ***a*** is the ***y*-intercept** and ***b*** is the **base** or **multiplying factor**. Sometimes *b* is known as the **growth factor**.

***Exercise #2*:** Consider the function $g(x) = 54\left(\frac{1}{3}\right)^x$

(a) Evaluate $g(0)$. What point does this indicate on the graph of g?

(b) Without the use of your calculator, determine the values of $g(1)$ and $g(2)$.

(c) Using your graphing calculator, sketch a graph of this function using the ***WINDOW*** $-2 \le x \le 4$ and $-10 \le y \le 100$. Mark the y-intercept.



(d) Why is this exponential function always decreasing while the one in Exercise #1 is always increasing?

INCREASING VS. DECREASING EXPONENTIALS

$y = a(b)^x$ will **increase** if _____

$y = a(b)^x$ will **decrease** if _____

***Exercise #3*:** For each of the following exponential functions, give its y-intercept and tell whether it is increase or decreasing.

(a) $y = 8\left(\frac{2}{3}\right)^x$ (b) $f(x) = 125(1.5)^x$ (c) $P(t) = 56\left(\frac{3}{2}\right)^x$

The equations of exponential functions are relatively easy to determine, if you understand this lesson so far. See what you can do in the next exercise.

***Exercise #4*:** Find the equation of the exponential function, in $y = a(b)^x$ form, for the function given in the table below. Show or explain your thinking.

x	0	1	2	3	4
y	10	30	90	270	810

Homework

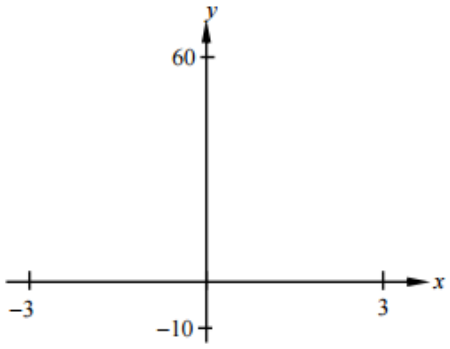
1.) Consider the exponential function $f(x) = 10(2)^x$

(a) Find the value of $f(0)$. What point does this represent on the graph of $y = f(x)$?

(b) Is this an increasing or decreasing exponential function? How can you tell based on its equation?

(c) Is this function's average rate of change over the interval $-1 \leq x \leq 2$ greater or less than that of the linear function $g(x) = 10x + 7$? Justify.

(d) Using your calculator, sketch a graph of this function on the axes shown below. Use the window indicated. Mark the y-intercept.



____ 2.) Which of the following is a decreasing exponential function whose y-intercept is 20?

[1] $y = 20\left(\frac{4}{3}\right)^x$

[2] $y = 20\left(\frac{1}{2}\right)^x$

[3] $y = -2x + 20$

[4] $y = \left(\frac{1}{3}\right)^x + 20$

____ 3.) Which of the following functions would best describe the data in the table?

[1] $y = 10x + 2$

[2] $y = 8x + 2$

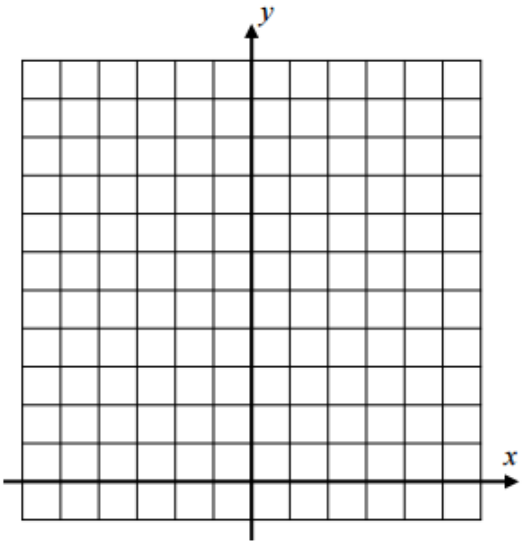
[3] $y = 5(2)^x$

[4] $y = 2(5)^x$

x	0	1	2	3	4
y	2	10	50	250	1250

4.) Graphing a basic exponential can be challenging because of how quickly they grow (or decay). In this exercise, we will graph one of the most basic.

$$f(x) = 2^x$$



(a) Evaluate each of the following and state the coordinate point that occurs on the graph of $f(x)$ based on the calculation.

$f(0) = 2^0 = 1$ $\quad$ $f(1) =$

$f(0) = 1$

$(0,1)$

$f(2) =$ $\quad$ $f(3) =$

(b) Evaluate each of the following. Remember your facts about negative exponents and give the point on the graph of $f(x)$.

$f(-1) =$ $\quad$ $f(-2) =$ $\quad$ $f(-3) =$

(c) Using the points you found in (a) and (b), graph this function for the domain interval $-3 \leq x \leq 3$.

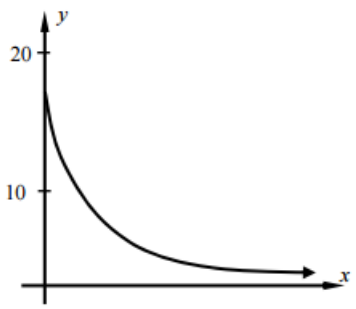
5.) Classify each of the following exponential functions as either increasing or decreasing and give the value of their y-intercepts.

(a) $y = 125(1.25)^x$ $\quad$ (b) $y = 22\left(\frac{3}{4}\right)^x$ $\quad$ (c) $y = 256\left(\frac{5}{2}\right)^x$

____ 6.) Which of the following could be the equation of the exponential function shown graphed below?

[1] $y = 15(1.25)^x$ $\quad$ [3] $y = 50(1.04)^x$

[2] $y = 18(0.75)^x$ $\quad$ [4] $y = 40(0.45)^x$



Review Section:

____ 7.) The table below shows the average yearly balance in a savings account where interest is compounded annually. No money is deposited or withdrawn after the initial amount is deposited.

Year	Balance, in Dollars
0	380.00
10	562.49
20	832.63
30	1232.49
40	1824.39
50	2700.54

Which type of function best models the given data?

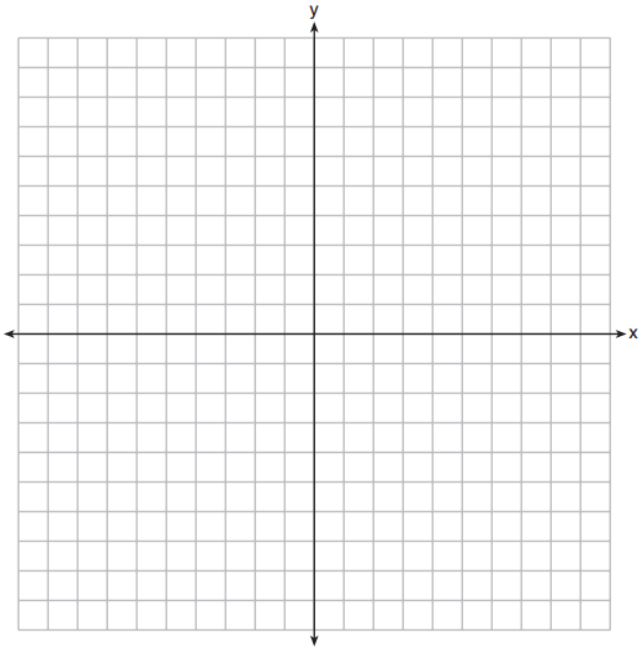
(1) linear function with a negative rate of change

(2) linear function with a positive rate of change

(3) exponential decay function

(4) exponential growth function

8.) Draw the graph of $y = \sqrt{x} - 1$ on the set of axes below.



Homework Answers

1.) a) $f(0) = 10$ $(0,10)$ This point is the y-intercept.

b) This is an increasing exponential function because its base value, b = 2, is greater than 1.

c) Average Rate of Change: $f(x) = 11\frac{2}{3}$ $g(x) = 10$
The function $f(x)$ has a greater average rate of change over this interval.

d) graph

2.) 2

3.) 4

4.) a) $f(0) = 1$ $f(1) = 2$ $f(2) = 4$ $f(3) = 8$

b) $f(-1) = \frac{1}{2}$ $f(-2) = \frac{1}{4}$ $f(-3) = \frac{1}{8}$

c) Graph

5.) a) Increasing function y-intercept = 125

b) Decreasing function y-intercept = 22

c) Increasing function y-intercept = 256

6.) 2

7.) 4

8.) Graph