Name ______________________________ Lesson #3 Date:______________

Factoring Polynomials Using Common Factors
Common Core Algebra 1

Factoring expressions is one of the gateway skills necessary for much of what we do in algebra for the rest of the course. The word **FACTOR** has two meanings and both are important.

<u>The two meanings of Factor</u>

1. <u>Factor (verb)</u> : To rewrite an algebraic expression as an **equivalent product**
2. <u>Factor (noun)</u> : An algebraic expression that is one part of a larger factored expression.

<u>Exercise #1</u>: Consider the expression $6x^2 + 15x$

(a)Write the individual terms $6x^2$ and $15x$ as completely factored expressions. Determine their **greatest common factor (GCF).**

(b) Using the Distributive Property, rewrite $6x^2 + 15x$ as a product involving the **GCF** from part (a).

(c) Evaluate both $6x^2 + 15x$ and your answer from part (b) for $x = 2$. What did you find? What does this support about the two expressions?

It is important that you are **fluent in reversing the distributive property** through using the greatest common factor **(GCF)**. The coefficient component of the GCF is the largest number that goes into all the terms evenly. *If all the terms have a common variable, the GCF is the variable with the lowest exponent.*

Exercise #2: For each of the following sets of monomials, identify the greatest common factor of each. Write each term as an extended product. The 1st example is completed for you.

(a) $12x^3$ and $18x$

GCF= **6x**

$12x^3 = \mathbf{6x}(2x^2)$ and $18x = \mathbf{6x}(x)$

(b) $5x^4$ and $25x^2$

(c) $21x^2y^5$ and $14xy^7$

(d) $24x^3$, $16x^2$, and $8x$

(e) $20x^2$, $-12x$, and 28

(f) $18x^2y^2$, $45x^2y$, and $90xy^2$

If you can correctly find the GCF for of a set of monomials you can easily transfer that skill and the Distributive Property to write equivalent factored expressions.

Exercise #3: Write each polynomial below as a factored expression involving the GCF of the polynomial. You then check your answer by distributing the GCF.

(a) $6x^2 + 10x$
$2x(3x + 5)$
Check:
$2x(3x + 5)$
$6x^2 + 10x$

(b) $3x - 24$

(c) $10x^2 - 15x$

(d) $4x^2 + 8x + 24$

(e) $6x^3 - 8x^2 + 2x$

(f) $10x^3 - 35x^2$

(g) $10x^2 - 40x - 50$

(h) $10x^4 - 2x^2$

(i) $8x^3 + 24x^2 - 32x$

Being able to **fluently factor out a GCF** is an essential skill. Sometimes a GCF is more than just a monomial. We have done this type of factoring back in Unit #1.

Exercise #4: Rewrite each expression as a product of two binomials by factoring out a common binomial factor.

(a) $(x+5)(x-1) + (x+5)(2x-3)$

(b) $(2x-1)(2x+7) - (2x-1)(x-3)$ ****Be careful w/ subtraction!***

Homework

1) Identify the GCF for each of the following sets of monomials:

(a) $6x^2$ and $24x^3$ (b) $15x$ and $10x^2$ (c) $2x^4$ and $10x^2$

(d) $2x^3$, $6x^2$, and $12x$ (e) $16t^2$, $48t$, and 80 (f) $8t^5$, $12t^3$, and $16t$

2) Which is the greatest common factor of the terms $36x^2y^4$ and $24x^7$

(1) $12xy^4$ (2) $24x^2y^7$ 3) $6x^2y^3$ (4) $3xy$

3) Write each of the following as equivalent products of the polynomials greatest common factor with another polynomial (***with the same number of terms***). The first one is done as an example:

(a) $8x – 28$
= 4(2x – 7)

b) $50x + 30$ (c) $24x^2 + 32x$

(d) $18 – 12x$ (e) $6x^3 + 12x^2 – 3x$ (f) $x^2 – x$

(g) $10x^2 + 35x – 20$ (h) $21x^3 – 14x$ (i) $36x – 8x^2$

(j) $30x^3 – 75x^2$ (k) $-16t^2 + 96t$ (l) $4t^3 – 32t^2 + 12t$

4) Which is ***not*** a correct <u>factorization</u> of the binomial $10x^2 + 40x$?

(1) $10x(x+4)$ (b) $10(x^2 + 4x)$ c) $5x(2x + 4)$ d) $5x(2x+8)$

5) Rewrite each of the fowling expressions as the product of two binomials by factoring out a common binomial factor. Watch out for the subtraction problems (b) and (d).

(a) $(x + 5)(x + 1) + (x + 5)(x + 8)$

(b) $(2x – 1)(3x + 5) – (2x – 1)(x + 4)$

(c) $(x – 7)(x – 9) – (x – 7)(4x + 5)$

(d) $(x + 1)(5x – 7) – (x + 1)(x – 3)$

Applications

6) The area of a rectangle is represented by $16x^2 + 56x$. The width of the rectangle is given as $2x + 7$.

(a) Give a monomial expression in terms of x for your length of the rectangle. Show how you arrived at your answer.

(b) If the length of the rectangle is 80, what is the width of the rectangle. Explain how you arrived at your answer.

Reasoning

7) These crazy polynomials keep acting like integers. We can factor integers to determine their factors. We can do the same with polynomials.

(a) List all the POSITIVE factors of 12 by writing them as integer products (such as $12=3\cdot4$)

(b) Write all of the factors of $2x^2 – 6x$ by also by writing all possible products [such as $2x^2-6x =2(x^2 -3x)$]

8) Which of the following is ***not*** a factor of $4x^2 + 12x$

(1) $x + 3$ (2) x (3) $3x$ (4) 4

Name ______________________________ Lesson #4 Date:____________

Factoring Based of Conjugate Pairs
Common Core Algebra 1

There are a number of different factoring techniques. But , each one of them boils down to reversing a product. We begin today by looking at products of conjugate binomials → (a+b)(a-b)

<u>Exercise #1</u>: Find each of the following products of conjugate pairs. See if you can work out a pattern.

(a) $(x + 5)(x - 5)$

(b) $(x – 2)(x + 2)$

(c) $(4x + 1)(4x – 1)$

(d) $(x + y)(x – y)$

(e) $(2x + 3)(2x – 3)$

(f) $(5x - 2y)(5x + 2y)$

The pattern we should see is that if we multiply conjugates, we end up with a ***binomial*** rather than a the expected ***trinomial.***

Multiplying Conjugate Pairs

$$(a + b)(a – b) = a^2 – b^2$$

<u>Exercise #2:</u> Use the pattern from ***<u>Exercise #</u>***1 to quickly rewrite the following products.

(a) $(x + 6)(x - 6)$

(b) $(5x – 2)(5x + 2)$

(c) $(2x + 7y)(2x – 7y)$

(d) $(4 + x)(4 – x)$

(e) $(6 + 5y)(6 – 5y)$

(f) $(10x - 4y)(10x + 4y)$

We now should be able to reverse this multiplication in order to rewrite the expressions that are the **<u>Difference Of Two Squares</u>** (D.O.T.S.) into products.

<u>Exercise #3:</u> Write each pf the following first in the form of $a^2 - b^2$ and then as equivalent products of conjugate pairs.

(a) $x^2 - 81$ (b) $9x^2 - 4$ (c) $25 - y^2$

(d) $4x^2 - 81y^2$ (e) $121x^2 - 1$ (f) $1 - 4x^2$

Never forget that when we factor, we are always rewriting an expression in a form that might look different, but it is ultimately still **<u>equivalent to the original</u>**.

<u>Exercise #4</u>: Let's look at the binomial $x^2 - 9$.

(a) Amelia believes that $x^2 - 9$ can be factored into $(x + 1)(x - 9)$ while her friend Isabel believes that is factored as $(x - 3)(x + 3)$. Fill out the table below to try to find evidence as to who is correct. *Use your calculator to help you.*

		Amelia	Isabel
x	$x^2 - 9$	$(x + 1)(x - 9)$	$(x - 3)(x + 3)$
0			
1			
2			
3			
4			

(b) By multiplying out their respective factors, show which of the two friends has the correct factorization. *Use the distributive property twice.*

Amelia: $(x + 1)(x - 9)$ Isabel: $(x - 3)(x + 3)$

Homework

1. Use the product of conjugates pattern, $\boldsymbol{(a + b)(a - b) = a^2 - b^2}$, to quickly find the following products in standard form.

(a) $(x + 7)(x - 7)$

(b) $(x - 10)(x + 10)$

(c) $(2 - x)(2 + x)$

(d) $(3x + 2)(3x - 2)$

(e) $(4x - 3)(4x + 3)$

(f) $(2x - 1)(2x + 1)$

(g) $(5 - 4x)(5 + 4x)$

(h) $(x^2 + 2)(x^2 - 2)$

(i) $(x^3 + 4)(x^3 - 4)$

2) Write each of the following binomials as an equivalent product of conjugates.

(a) $x^2 - 16$

(b) $x^2 - 100$

(c) $x^2 - 1$

(d) $x^2 - 25$

(e) $4 - x^2$

(f) $9 - x^2$

(g) $4x^2 - 1$

(h) $16x^2 - 49$

(i) $1 - 25x^2$

(j) $4x^2 - 9y^2$

(k) $81 - 4t^2$

(l) $x^4 - 36$

Applications

3) A square is changed into a new rectangle by increasing it's width by two inches and decreasing its length by 2 inches. ***Make sure to draw a picture (or pictures) to help you solve these problems.***

(a) If the original square had a side length of 8 inches, find its area and the area of the new rectangle. How many square inches larger is the square's area?

(b) If the original square had a side length of 20 inches, find its area and the area of the new rectangle. How many square inches larger is the square's area?

(c) If the square had a side length of x inches, show that its area will always be 4 more square inches than the area of the new rectangle.

Reasoning

4) Consider the numerical expression $51^2 – 49^2$.

(a) Use your calculator to find the numerical value of this expression.

(b) Can you use the facts about conjugate pairs to Show why this difference should work out to be the answer from (a)?

Reasoning

5) Consider the following expression (x + 2)(x – 2) – (x + 4)(x – 4)

(a) Using your calculator, determine the value of this expression for the various values of x.

(b) Algebraically, show that this product has a constant value [see your answer in (a)] regardless of the value of x.

x	(x + 2)(x - 2) - (x + 4)(x - 4)
-2	
-1	
0	
1	
2	

Name ____________________________ Lesson #5 Date:________________

Factoring Trinomials
Common Core Algebra 1

So far we have worked on two factoring techniques (1) Factoring based on Common Factors (**GCF)** and (2) Factoring Based on Conjugate Pairs (**DOTS).** Today we will tackle the most difficult of the factoring techniques, the *Factoring of Trinomials.* Before we factor trinomials, let's review multiplying binomials.

Exercise #1: Write each of the following products is Standard Form.

(a) $(x + 5)(x + 3)$ (b) $(x – 2)(x + 7)$ (c) $(x – 5)(x – 10)$

(d) $(2x – 3)(5x + 1)$ (e) $(6x + 7)(x + 2)$ (f) $(4x – 1)(2x – 5)$

Now we need to reverse this process to take a trinomial and write it as the product of two binomials. We want to use patterns found in finding the product of two binomials. We will also use factoring using the GCF (as a binomial) like we did in Lesson 3.

Factor: $x^2 + 5x – 24$

1) Look for a GCF:
   a. There is no GCF for this trinomial
   b. The only way this method works is if you take out the GCF (if there is one.)

2) Take the coefficient for x^2 (1) and multiply it with the last term (24):

$x^2 + 5x – 24$ $1 \cdot 24 = 24$

$x^2 + 8x – 3x – 24$ * Now find factors of 24 with a sum of +5. The numbers will be +8 and -3, which become 8x and 3x. The terms must be +8x and -3x (because they have a sum of +5x)

3) **SPLIT THE MIDDLE** and reduce each side:

$x^2 + 8x \mid – 3x – 24$
Take Out: x and -8
$x(x + 8) - 3(x + 8)$

*When you're done the binomial on each side should be the same.

4) Take out the common binomial (x + 8) as a GCF, and you are left with x on the left and – 3 on the right. They make up the binomial (x – 3)

5) Your binomial factors are (x + 8) and (x - 3)

6) Check: $(x + 8)(x – 3)$

$x(x - 3) + 8(x – 3)$
$x^2 – 3x + 8x – 24$
$x^2 + 5x – 24$ (It checks!!)

Exercise #2: Let's take the same binomial, $x^2 + 5x – 24$ and see what would happen if we switched the order of the terms in Step 2. Finish the last 3 steps.

As done above:	Finish the steps:
$x^2 + 5x – 24$	$x^2 + 5x - 24$
$x^2 + \boldsymbol{8x – 3x} – 24$	$x^2 \boldsymbol{– 3x + 8x} – 24$

Exercise #3: Find the 2 binomial factors of $x^2 – 22x + 40$. Check by finding the product of your two binomial factors.

Exercise #4: Find the 2 binomial factors of $x^2 – x - 56$. Check by finding the product of your two binomial factors.

<u>Exercise #5:</u> So far the examples we have factored have all had a coefficient of 1 for x^2. Let's see if the process works when the coefficient for x^2 is a number other than 1. Find the binomial factors of each trinomial and check.

Factor: $3x^2 - x – 24$

1) Look for a GCF:
   a. There is no GCF for this trinomial
   b. The only way this method works is if you take out the GCF (if there is one.)

2) Take the coefficient for x^2 (3) and multiply it with the last term (24):

$3x^2 - x – 24$	$3 \cdot 24 = 72$
$3x^2 + 8x – 3x – 72$	* * Now find factors of 72 with a sum of -1. The numbers will be -9 and +8, which become -9x and +8x. The terms must be -9x and +8x (because they have a sum of -x, or -1x)

3) **SPLIT THE MIDDLE** and reduce each side:

$3x^2 - 9x \mid + 8x – 24$
Take Out: 3x and +8
$3x(x - 3) + 8(x - 3)$

*When you're done the binomial on each side should be the same.

4) Take out the common binomial (x - 3) as a GCF, and you are left with 3x on the left and +8 on the right. They make up the binomial (3x+ 8)

5) Your binomial factors are (x-3) and (3x + 8)

6) Check: $(x – 3)(3x + 8)$

$x(3x + 8) - 3(3x + 8)$
$3x^2 + 8x – 9x – 24$
$3x^2 - x – 24$ (It checks!!)

a) $6x^2 – 5x – 4$ b) $8x^2 + 7x – 1$ c) $12x^2 – 20x + 7$

Homework

1) Which of the following products is equivalent to $x^2 - 10x - 24$

a) $(x - 6)(x + 4)$ b) $(x + 6)(x - 4)$ c) $(x - 12)(x + 2)$ d) $(x + 12)(x - 2)$

2) Which of the following products is equivalent to $x^2 - 17x + 60$

b) $(x + 12)(x + 5)$ b) $(x - 12)(x - 5)$ c) $(x - 20)(x + 3)$ d) $(x + 20)(x - 3)$

3) Which of the following products is equivalent to $3x^2 - 2x - 5$

a) $(x - 5)(x + 3)$ b) $(3x + 5)(x - 1)$ c) $(x + 1)(3x - 5)$ d) $(3x + 3)(3x - 5)$

4) Write each of the following trinomials in its equivalent factored form. Make sure each answer is correct by checking. Don't give up on any problem… keep trying… use extra scrap paper as needed! (Level: Easy)

a) $x^2 + 10x + 15$ b) $x^2 + 12x + 35$ c) $x^2 + 12x + 36$

d) $x^2 - 5x + 6$ e) $x^2 - 9x + 20$ f) $x^2 - 12x + 20$

g) $x^2 + 3x - 18$ h) $x^2 + 3x - 40$ i) $x^2 + x - 30$

j) $x^2 - 5x - 50$ k) $x^2 - x - 42$ l) $x^2 - 4x - 60$

5) Write each of the following trinomials in its equivalent factored form. Make sure each answer is correct by checking. We're going to raise the level of difficulty….. use extra scrap paper as needed! (Level: Medium)

a) $2x^2 + 13x + 21$

b) $5x^2 – x – 6$

c) $3x^2 – 16x – 12$

d) $7x^2 + 11x – 6$

e) $2x^2 – x – 10$

f) $5x^2 – 31x + 6$

6) We’re going to raise the level of difficulty one more time. Write each of the following trinomials in its equivalent factored form. Make sure each answer is correct by checking. Use extra scrap paper as needed! (Level: Hard)

a) $4x^2 + 27x + 18$ b) $21x^2 – 19x + 4$ c) $12x^2 – 31x + 20$

d) $24x^2 – x – 5$ e) $12x^2 + 53x – 14$ f) $32x^2 – 20x – 7$

Applications:

7) A rectangle has been shown in terms of a variable, x:

Width = ?

Area = $2x^2 + 9x + 4$

length = $x + 4$

a) Find a binomial expression that represents the rectangle's width. Justify your answer using the rectangle's width and area.

b) If the width of the rectangle is 21, what are the length and area? Show all work and use appropriate units.

Name ________________________ Lesson #6 Date:______________

The Complete Factoring of Trinomials
Common Core Algebra 1

Factoring trinomials is an extremely important skill. It is a skill that must be mastered, no matter how difficult the polynomials become. What we need to do in this lesson is combine a few of our factoring methods to learn a process called **complete factoring.**

Exercise #1: Consider the trinomial $4x^2 + 20x + 24$

(a) Write this trinomial as an equivalent expression involving the product of the term's GCF and another trinomial.

(b) Factor this additional trinomial to express original in **completely factored form.**

Whenever we factor, we should always look to see if a greatest common factor (**GCF**) exists than can be factored out to begin the problem. Taking out a GCF 1st makes the subsequent factoring easier as it makes the coefficients and/or the exponents smaller.

Exercise #2: Completely Factor each trinomial.

(a) $10x^2 + 15x - 10$

(b) $3x^3 - 21x^2 + 36x$

(c) $7x^2 + 21x - 70$

(d) $6x^2 - 2x - 4$

Complete factoring can also involve **Difference of Two Squares (**DOTS) problems. Try the next exercise to see how this works.

<u>Exercise #3</u>: Write each of the following binomials in completely factored form.

(a) $2x^2 – 18$ (b) $5x^3 – 20x$ (c) $12x^2 – 3$

(d) $54x^2 – 24$ (e) $100x^2 – 25$ (f) $81x^4 – 36x^2$

The true test of fluency in complete factoring is testing if you have the ability to factor a polynomial without being told what type of polynomial it is.

<u>Exercise #</u>4: Write each of the following polynomials in its completely factored form.

(a) $x^2 – 8x - 20$ (b) $9x^2 – 49$ (c) $5x^2 + 5x - 100$

(d) $6x^2 – 150$ (e) $7x^2 – 49x + 84$ (f) $7x^2 + 36x + 5$

Homework

1. Rewrite each of the following trinomials in completely factored form:

(a) $2x^2 + 20x + 42$

(b) $6x^2 + 33x + 15$

(c) $5x^2 – 10x – 40$

(d) $30x^2 + 20x – 10$

(e) $x^3 + 7x^2 + 10x$

(f) $4x^3 + 10x^2 – 24x$

(g) $5x^2 – 45$

(h) $2x^3 – 2x$

(i) $36 – 4x^2$

(j) $20x^2 – 125$

(k) $12x^2 – 10x – 10$

(l) $12x^2 - 19x + 4$

2) Which of the following is *not* a factor of $4x^3 + 12x^2 - 72x$? Show your work to justify your choice.

(a) $(x + 9)$ (b) $(x - 3)$ (c) $4x$ (d) $(x + 6)$

3) Which of the following is the missing factor in the product $2(x -1)(\ \ ?\ \ )$ if it is equivalent to the trinomial $2x^2 + 10x - 12$?

(1) $x + 12$ (2) $x + 6$ (3) $x + 3$ (4) $x - 5$

4) Rewrite each of the following trinomials in completely factored form:

(a) $40x^2 - 90$ (b) $40x^2 - x - 3$ (c) $40x^2 + 60x - 100$

(d) $40x^2 - 440x + 920$ (e) $40x^{11} - 40$ (f) $40x^2 + 21x + 2$

Reasoning

5) Consider the cubic trinomial $x^3 + 8x^2 + 7x$

(a) Write this trinomial as an equivalent product in completely factored form.

(b) How can the original trinomial and your answer to (a) help you determine the value of (10)(17)(11) without a calculator?

6) Consider the cubic trinomial $x^3 – 8x^2 - 48x$

(a) Write this trinomial as an equivalent product in completely factored form.

(b) How can the original trinomial and your answer to (a) help you determine the value of (10)(14)(-2) without a calculator?

7) Use the complete factorization of $2x^3 + 8x^2 + 8x$ to determine the product of $(20)(12)^2$. Explain your reasoning.

8) Use the complete factorization of $2x^3 + 8x^2 + 8x$ to determine the product of $(20)(12)^2$. Explain your reasoning.

Mixed Factoring Extra Practice:

1) $7x^4 - 28x^3 - 224x^2$

2) $x^2 + 3x + 54$

3) $x^2 - 40x + 364$

4) $6x^2 - 486$

5) $9x^2 + 144x + 576$

6) $225x^2 - 25$

7) $x^2 + 2x - 1{,}023$

8) $8x^2 + 14x - 4$

9) $8x^2 - 40x - 288$

10) $8x^2 - 15x + 7$

11) $3x^2 - 3x + 90$

12) $x^2 - 9x + 20$

13) $x^2 - 121$

14) $x^2 - 13x - 30$

15) $x^2 - 13x - 48$

16) $4x^2 + 4x - 288$

17) $20x^2 - 21x - 5$

18) $20x^2 - 60x - 1{,}080$

19) $20x^2 + 35x - 10$

20) $24x^2 - 73x + 3$

21) $24x^2 - 168x - 1{,}056$

22) $24x^2 + 10x - 6$

23) $36x^2 - 26x + 4$

24) $36x^2 - 252x - 1{,}584$

25) $36x^2 + 2x - 5$

26) $24x^2 - 73x + 3$

27) $24x^2 - 168x - 1{,}056$

28) $24x^2 + 10x - 6$

29) $4x^2 - 121$

30) $9x^2 - 196$

31) $16x^2 - 100$

32) $49x^2 - 196$

33) $144x^2 - 100$

34) $5x^2 - 320$

35) $10x^2 - 810$

36) $20x^2 - 245$

37) $169x^2 - 121$

38) $7x^2 - 112$

39) $25x^2 - 256$

40) $25x^2 - 225$

41) $11x^2 - 1{,}331$

42) $6x^2 - 600$

43) $81x^2 - 16$

44) $100x^2 - 1$

45) $16x^2 - 49$

46) $16x^2 - 144$

47) $16x^2 - 36$

48) $16x^2 - 9$

49) $225x^2 - 144$

50) $225x^2 - 25$

51) $225x^2 - 100$

52) $225x^2 - 400$

More Extra Practice:

1) $x^2 - 5x - 60$

2) $x^2 + 11x - 30$

3) $12x^2 - 28x - 5$

4) $x^2 - 19x + 48$

5) $3x^2 - 3x - 216$

6) $18x^2 + 6x - 4$

7) $x^2 - 121$

8) $24x^2 + 22x + 5$

9) $x^2 - x + 30$

10) $144x^2 - 36$

11) $5x^9 - 320x^7$

12) $4x^2 + 9$

13) $x^2 – 15x - 54$

14) $14x^2 – 20x + 6$

15) $x^2 + 45x + 506$

16) $x^2 + 7x – 30$

17) $8x^3 + 24x^2 + 16x$

18) $x^2 – 256$

19) $x^2 – 2x – 1,098$

20) $x^2 – 30x + 396$

21) $x^2 + 5x – 4$

22) $x^2 + x – 3,080$

23) $15x^2 – 105x + 90$

24) $15x^2 + 27x + 12$