Physics  

Week 15(Sem. 2)  

The Atom Chapter Summary  

**Line Spectra**  

From the last section, we know that all objects emit electromagnetic waves. For a solid object, such as the filament of a light bulb, these waves have a continuous range of wavelengths, some of which are in the visible region. On the other hand, individual atoms, free of strong interactions that are present in the solid, emit only certain specific wavelengths, rather than a continuous spectrum. Therefore, the study of individual atoms is carried out using gasses at low pressure (such that the atoms are very far apart). The gases can be promoted to emit electromagnetic waves by applying a large potential difference between two electrodes located within the tube. Using a diffraction grating, a series of bright fringes (lines) appear. The resulting series of lines is called a line spectrum or bright line spectrum, this is because each fringe appears as a thin rectangle. The simplest line spectrum is produced by the element Hydrogen (H). The figure below shows Hydrogens’ bright line spectrum along with some more complicated ones like Ne and Hg.

The collection of lines that are in the visible region for atomic hydrogen has been named the Balmer series. The equations below can be applied to each series, from Lyman, to Balmer, to Paschen.

Lyman series \( \frac{1}{\lambda} = R \left( \frac{1}{1^2} - \frac{1}{n^2} \right) \) \( n=2,3,4,... \)

Balmer series \( \frac{1}{\lambda} = R \left( \frac{1}{2^2} - \frac{1}{n^2} \right) \) \( n=3,4,5,... \)

Paschen series \( \frac{1}{\lambda} = R \left( \frac{1}{3^2} - \frac{1}{n^2} \right) \) \( n=4,5,6,... \)

For the above equations the constant \( R=1.097\times10^7 \) m\(^{-1}\) and is called the Rydberg constant. Notice that with each group of lines there is a short and long wavelength limit; also the number of lines increase toward the short wavelength side. It is also important to note that the equations provide wavelengths, but no reason why certain wavelengths. See example 2.

**Bohr Model**  

Adopting Planck’s idea of quantized energy levels, Bohr hypothesized that in a hydrogen atom there can only be certain values of the total energy(KE & PE). This allowed for levels of differing energy, larger orbits having larger energy. Bohr said electrons don’t just radiate waves, only when an electron in an initial orbit waith a larger energy \( (E_i) \) changes to a final orbit with a smaller energy. The electron was risen to this higher energy level by either heat or direct voltage electricity. Therefore, the change is energy is equal to \( hf \) (OR \( E_f-E_i = hf \)). Where \( f = c/\lambda \) for light waves.
If the total energy of the atom were to be analyzed it would be KE + PE = E. With some background and derivation one can arrive at the equation for the radii for Bohr’s orbits (in meters):

\[ r_n = \left(\frac{5.29 \times 10^{-11}}{n^2}\right) \frac{n^2}{Z} \]

Where \( r_n \) is the radius, \( Z=1 \) for Hydrogen, and \( n=1 \) for the smallest Bohr orbit. Therefore, for hydrogen the smallest Bohr radii is \( r_1 = 5.29 \times 10^{-11} \) m. and is called the Bohr radius. Using this equation and the energy equation, Bohr energy levels in Joules will be

\[ E_n = -\frac{(2.18 \times 10^{-18}) Z^2}{n^2} \quad n=1,2,3,\ldots \]

Or the same equation in electron volts

\[ E_n = -\left(13.6\right) \frac{Z^2}{n^2} \quad n=1,2,3,\ldots \]

**Energy Level Diagrams**

For a hydrogen atom, the lowest energy level, \( n=1 \), corresponds to an energy of \(-13.6\) eV. This lowest energy state is called the ground state, higher levels are called the excited states. The highest energy state would be \( n=\infty \). Notice the energies become closer together as \( n \) increases. Thus the amount of energy needed to raise the electron from the ground state \((n=1)\) to the excited state \((n=\infty)\) would be \(13.6\) eV. Supplying this energy would remove the electron from hydrogen producing a positive ion (H+), this is called the ionization energy. See Example 3 for Lithium’s ionization.

**Spectral Lines for Hydrogen**

The Lyman series occurs when electrons make transitions from higher energy levels with \( n_i = 2, 3, 4,\ldots \) to the first energy level \( n_f = 1 \). For the Balmer series the electron transition is from \( n_i = 3, 4, 5,\ldots \) to \( n_f = 2 \), while the Paschen series is from \( n_i = 4, 5, 6,\ldots \) to \( n_f = 3 \). See Example 4.

**Quantum Mechanical – Hydrogen Atom**

This model involves quantum mechanics and the Schrödinger equation, it differs in many ways from the Bohr model. Because \( n \) is chosen as a discrete value (integer) it can have a continuous range and is called a quantum number. In contrast, quantum mechanics reveals that there are 4 numbers that are needed to describe each electron in the atom.

1. Principle Quantum Number \((n)\) – this number determines the total energy of the electron.
2. Orbital Quantum Number \((\ell)\) – determines the angular momentum of the electron, \(\ell = 0, 1, 2, \ldots n-1\). For \(n=1\), then \(\ell\) only equals 0.
3. Magnetic Quantum Number \((m)\) – Zeeman effect, when an external magnetic field is applied it influences the energy of the atom. \(m_\ell = -\ell, -\ell+1, \ldots, 0, 1, \ldots, \ell\).
4. Spin Quantum Number \((m_s)\) – Analogous to the way the earth spins around the sun, the electrons spin around the nucleus. Therefore
they can spin two ways +1/2 or -1/2 (up or down).

See Example 5 for Hydrogens’ possible states.

**Electron Probability Clouds**

According to the Bohr’s model, the nth orbit is a circle of radius \( r_n \), and every time the position of the electron in this orbit is measured, the electron is found exactly a distance \( r_n \) away from the nucleus. This is too simple and known to be untrue. For \( n=1 \), the position is uncertain in the sense that there is a cloud of probability of finding an electron sometimes near to the nucleus and sometimes far away. The probability is determined by the wave function \( \psi \). If one were to put dots everywhere an electron was found there would be a cloud of dots with the highest density region being the darkest. This highest probability region corresponds to \( 5.29 \times 10^{-11} \text{m} \), the same as Bohr’s findings.

For a principle quantum number of 2, the probability clouds are different than for \( n=1 \). There is actually more than one cloud shape because \( \ell \) can equal 0 or 1, the \( \ell \)

doesn’t effect energy but does effect cloud shape. For \( \ell \) equal to 0 the shape is a basic sphere and for \( \ell \) equal to 1 it is called a dumbbell shape.

**Pauli Exclusion Principle**

Except for hydrogen, all electrically neutral atoms contain more than one electron, the number being given by the atomic number (Z) for the element. Pauli’s exclusion principle says “that no two electrons in an atom can have the same set of values for the four quantum numbers, \( n, \ell, m, m_s \).” They can have any combination of numbers such that they aren’t identical, so the last number can be the only difference.

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**EXAMPLE 2 • The Balmer Series**

Find (a) the longest and (b) the shortest wavelengths of the Balmer series.

**Reasoning** Each wavelength in the series corresponds to one value for the integer \( n \) in Equation 30.2. Longer wavelengths are associated with smaller values of \( n \). The longest wavelength occurs when \( n \) has its smallest value of \( n = 3 \). The shortest wavelength arises when \( n \) has a very large value, so that \( 1/n^2 \) is essentially zero.

**Solution**

(a) With \( n = 3 \), Equation 30.2 reveals that for the longest wavelength

\[
\frac{1}{\lambda} = R \left( \frac{1}{2^2} - \frac{1}{n^2} \right) = (1.097 \times 10^7 \text{ m}^{-1}) \left( \frac{1}{2^2} - \frac{1}{3^2} \right)
\]

\[= 1.524 \times 10^6 \text{ m}^{-1} \text{ or } \lambda = 656 \text{ nm} \]

(b) With \( 1/n^2 = 0 \), Equation 30.2 reveals that for the shortest wavelength

\[
\frac{1}{\lambda} = (1.097 \times 10^7 \text{ m}^{-1}) \left( \frac{1}{2^2} - 0 \right) = 2.743 \times 10^6 \text{ m}^{-1} \text{ or } \lambda = 365 \text{ nm} \]
EXAMPLE 3 • The Ionization Energy of Li$^{2+}$

The Bohr model does not apply when more than one electron orbits the nucleus, because the model does not account for the electrostatic force that one electron exerts on another. For instance, an electrically neutral lithium atom (Li) contains three electrons in orbit around a nucleus that includes three protons ($Z = 3$), and Bohr's analysis is not applicable. However, the Bohr model can be used for the doubly charged positive ion of lithium (Li$^{2+}$) that results when two electrons are removed from the neutral atom, leaving only one electron to orbit the nucleus. Obtain the ionization energy that is needed to remove the remaining electron from Li$^{2+}$.

**Reasoning** The lithium ion Li$^{2+}$ contains three times the positive nuclear charge as that of the hydrogen atom. Therefore, the orbiting electron is attracted more strongly to the nucleus in Li$^{2+}$ than in the hydrogen atom. As a result, we expect that more energy is required to ionize Li$^{2+}$ than the 13.6 eV required for atomic hydrogen.

**Solution** The Bohr energy levels for Li$^{2+}$ are given by Equation 30.13 with $Z = 3$:
\[
E_n = -(13.6 \text{ eV})(\frac{3^2}{n^2}).
\]
Therefore, the ground state ($n = 1$) energy is
\[
E_1 = -(13.6 \text{ eV}) \frac{3^2}{1^2} = -122 \text{ eV}.
\]

To remove the electron from Li$^{2+}$, 122 eV of energy must be supplied: $\text{Ionization energy} = 122 \text{ eV}$. This value for the ionization energy agrees well with the experimental value of 122.4 eV and, as expected, is greater than the 13.6 eV required for atomic hydrogen.
EXAMPLE 4 • The Brackett Series for Atomic Hydrogen

In the line spectrum of atomic hydrogen there is also a group of lines known as the Brackett series. These lines are produced when electrons, excited to high energy levels, make transitions to the \( n = 4 \) level. Determine (a) the longest wavelength in this series and (b) the wavelength that corresponds to the transition from \( n_i = 6 \) to \( n_f = 4 \). (c) Refer to Figure 24.9 and identify the spectral region in which these lines are found.

**Reasoning** The longest wavelength corresponds to the transition that has the smallest energy change, which is between the \( n_i = 5 \) and \( n_f = 4 \) levels in Figure 30.11. The wavelength for this transition, as well as that for the transition from \( n_i = 6 \) to \( n_f = 4 \), can be obtained from Equation 30.14.

**Solution**

(a) Using Equation 30.14 with \( Z = 1, n_i = 5 \), and \( n_f = 4 \), we find that

\[
\frac{1}{\lambda} = (1.097 \times 10^7 \text{ m}^{-1})(1^2) \left( \frac{1}{4^2} - \frac{1}{5^2} \right)
\]

\[
= 2.468 \times 10^5 \text{ m}^{-1} \quad \text{or} \quad \lambda = 4051 \text{ nm}
\]

(b) The calculation here is similar to that in part (a):

\[
\frac{1}{\lambda} = (1.097 \times 10^7 \text{ m}^{-1})(1^2) \left( \frac{1}{4^2} - \frac{1}{6^2} \right)
\]

\[
= 3.809 \times 10^5 \text{ m}^{-1} \quad \text{or} \quad \lambda = 2625 \text{ nm}
\]

(c) According to Figure 24.9, these lines lie in the infrared region of the spectrum.
EXAMPLE 5 • Quantum Mechanical States of the Hydrogen Atom

Determine the number of possible states for the hydrogen atom when the principal quantum number is (a) \( n = 1 \) and (b) \( n = 2 \).

**Reasoning** Each different combination of the four quantum numbers summarized in Table 30.1 corresponds to a different state. We begin with the value for \( n \) and find the allowed values for \( \ell \). Then, for each \( \ell \) value we find the possibilities for \( m_\ell \). Finally, \( m_s \) may be \(+\frac{1}{2}\) or \(-\frac{1}{2}\) for each group of values for \( n \), \( \ell \), and \( m_\ell \).

**Solution**

(a) The diagram below shows the possibilities for \( \ell \), \( m_\ell \), and \( m_s \) when \( n = 1 \):

\[
\begin{array}{c}
\text{n} = 1 \\
\rightarrow \ell = 0 \\
\rightarrow m_\ell = 0 \\
\end{array}
\]

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Thus, there are two different states for the hydrogen atom. These two states have the same energy, since they have the same value of \( n \).

(b) When \( n = 2 \), there are eight possible combinations for the values of \( n \), \( \ell \), \( m_\ell \), and \( m_s \), as the diagram below indicates:

\[
\begin{array}{c}
\text{n} = 2 \\
\rightarrow \ell = 1 \\
\rightarrow m_\ell = 0 \\
\end{array}
\]

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With the same value of \( n = 2 \), all eight states have the same energy.
1. In the currently accepted model of the atom, a fuzzy cloud around a hydrogen nucleus is used to represent the 
(1) electron's actual path, which is not a circular orbit 
(2) general region where the atom's proton is most probably located 
(3) general region where the atom's electron is most probably located 
(4) presence of water vapor in the atom

2. A model of the atom in which the electrons can exist only in specified orbits was suggested by 
(1) Bohr 
(2) Planck 
(3) Einstein 
(4) Rutherford

3. Base your answer to the following question on the statement below. 
The spectrum of visible light emitted during transitions in excited hydrogen atoms is composed of blue, green, red, and violet lines. 
Which color of light in the visible hydrogen spectrum has photons of the shortest wavelength? 
(1) blue 
(2) green 
(3) red 
(4) violet

4. Which type of photon is emitted when an electron in a hydrogen atom drops from the \( n = 2 \) to the \( n = 1 \) energy level? 
(1) ultraviolet 
(2) visible light 
(3) infrared 
(4) radio wave

5. A hydrogen atom with an electron initially in the \( n = 2 \) level is excited further until the electron is in the \( n = 4 \) level. This energy level change occurs because the atom has 
(1) absorbed a 0.85-eV photon 
(2) emitted a 0.85-eV photon 
(3) absorbed a 2.55-eV photon 
(4) emitted a 2.55-eV photon

6. What is the minimum energy needed to ionize a hydrogen atom in the \( n = 2 \) energy state? 
(1) 13.6 eV 
(2) 10.2 eV 
(3) 3.40 eV 
(4) 1.89 eV

7. An electron in a hydrogen atom drops from the \( n = 3 \) energy level to the \( n = 2 \) energy level. The energy of the emitted photon is 
(1) 1.51 eV 
(2) 1.89 eV 
(3) 3.40 eV 
(4) 4.91 eV

8. The electron in a hydrogen atom drops from energy level \( n = 2 \) to energy level \( n = 1 \) by emitting a photon having an energy of approximately 
(1) \( 5.4 \times 10^{-19} \) J 
(2) \( 1.6 \times 10^{-18} \) J 
(3) \( 2.2 \times 10^{-18} \) J 
(4) \( 7.4 \times 10^{-18} \) J

9. An excited hydrogen atom returns to its ground state. A possible energy change for the atom is a 
(1) loss of 10.20 eV 
(2) gain of 10.20 eV 
(3) loss of 3.40 eV 
(4) gain of 3.40 eV

10. A photon having an energy of 15.5 electron volts is incident upon a hydrogen atom in the ground state. If the photon is absorbed by the atom, it will 
(1) ionize the atom 
(2) excite the atom to \( n = 2 \) 
(3) excite the atom to \( n = 3 \) 
(4) excite the atom to \( n = 4 \)

11. What is the minimum amount of energy required to ionize a hydrogen atom in the \( n = 2 \) state? 
(1) 13.6 eV 
(2) 10.2 eV 
(3) 3.4 eV 
(4) 0 eV

12. Compared to the total energy of the hydrogen atom in the ground state, the total energy of the atom in an excited state is 
(1) less 
(2) greater 
(3) the same

13. A hydrogen atom undergoes a transition from the \( n = 3 \) state to the ground state. The total number of different possible photon energies that may be emitted is 
(1) 1 
(2) 2 
(3) 3 
(4) 4
14. A hydrogen atom is in the \( n = 5 \) energy state after having absorbed a 0.97-eV photon. What was the original energy state of the hydrogen atom?

(1) \( n = 1 \)  
(2) \( n = 2 \)  
(3) \( n = 3 \)  
(4) \( n = 4 \)

15. The diagram below represents the bright-line spectra of four elements, \( A, B, C, \) and \( D \), and the spectrum of an unknown gaseous sample.

Based on comparisons of these spectra, which two elements are found in the unknown sample?

(1) \( A \) and \( B \)  
(2) \( A \) and \( D \)  
(3) \( B \) and \( C \)  
(4) \( C \) and \( D \)

16. Which electron transition in the hydrogen atom results in the emission of a photon of greatest energy?

(1) \( n = 2 \) to \( n = 1 \)  
(2) \( n = 3 \) to \( n = 2 \)  
(3) \( n = 4 \) to \( n = 2 \)  
(4) \( n = 5 \) to \( n = 3 \)

17. An excited atom emits a photon of energy \( E \) when an electron changes from energy level \( n = 3 \) to \( n = 2 \). In order for the same electron to change directly from energy level \( n = 2 \) to \( n = 3 \), it may

(1) absorb a photon with energy \( E \)  
(2) absorb a photon with energy \( 2E \)  
(3) emit a photon with energy \( 3E \)  
(4) emit a photon with energy \( E/2 \)