HIGH SCHOOL MATHEMATICS CONTESTS

Math League Press, P.O. Box 17, Tenafly, New Jersey 07670-0017

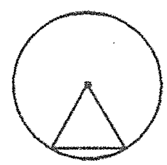
All official participants must take this contest at the same time.

Contest Number 3 *Any calculator without a QWERTY keyboard is allowed.* Answers must be exact *or* have 4 (or more) significant digits, correctly rounded. **December 3, 2013**

Name Answer Key **Teacher** ________ **Grade Level** ____ **Score** ____

Time Limit: 30 minutes NEXT CONTEST: JAN. 14, 2014 *Answer Column*

3-1. One vertex of an equilateral triangle is at the center of a circle, and the other two vertices lie on the circle, as shown. If the circle's area is 16π, what is the triangle's perimeter?



3-1. 12

3-2. My uncle's birthday is today. He is less than 100 years old, and his age is six times the sum of its digits. How old is he?

3-2. 54

3-3. The first 4 rows of an array of consecutive integers is shown at the right. The first row has 1 entry. Every other row has 2 more entries than the row directly above it. What is the value of the 2013th entry in the 2013th row? [**NOTE:** For this question, provide an exact answer, not an approximate answer.]

1
2 3 4
5 6 7 8 9
10 11 12 13 14 15 16

3-3. 4,050,157

3-4. If the third-degree polynomial equation $P(x) = 0$ has three unequal real roots, what is the least possible number of unequal real roots there could be for the sixth-degree polynomial equation $P(x^2) = 0$?

3-4. 0

3-5. In an *arithmetic sequence* (such as 7, 12, 17, 22, . . .), the difference between successive terms is fixed. If the sum of the 72nd and 112th terms of one such sequence is 22, what is the sum of the first 183 terms?

3-5. 2013

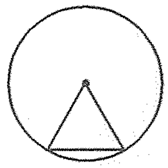
3-6. After five integers are paired in all possible ways, the integers in each pair are added. The ten sums obtained (not all different) are 1, 4, 5, 7, 8, 8, 11, 11, 14, and 15. What are these five integers?

3-6. -1, 2, 5, 6, 9

Contest # 3 *Answers & Solutions* 12/3/13

Problem 3-1

Since the area of the circle is 16π, the length of each radius of the circle is 4. Since the triangle is equilateral, its perimeter is $3 \times 4 = \boxed{12}$.



Problem 3-2

My uncle's age must be a two-digit number, so we can represent his age as $10t+u$, where t and u are one-digit numbers. We know that $10t+u = 6(t+u)$, from which we get $4t = 5u$. The only solution that meets the conditions of the problem is $(t,u) = (5,4)$, so my uncle's age is $\boxed{54}$.

Problem 3-3

The right-most entry in row n is the number n^2, so the final entry in row 2012 is 2012^2. Therefore, the 2013th entry in the next row, the 2013th row, is $2012^2 + 2013 = \boxed{4\,050\,157}$.

Problem 3-4

Consider the function $P(x) = (x+1)(x+4)(x+9)$, whose roots are the real numbers -1, -4, and -9. Since $P(x^2) = (x^2+1)(x^2+4)(x^2+9) > 0$ whenever x is real, and since $P(x^2) = 0$ has only imaginary roots, there is at least one such polynomial equation for which the number of real roots is $\boxed{0}$.

Problem 3-5

Method I: In our 183-term sequence a, $a+d$, $a+2d$, $a+3d$, . . . , $a+182d$, the 72nd term is $a+71d$, the 112th term is $a+111d$, and the middle term is $a+91d$. We're told that the sum of the 72nd term and 112th terms is 22, so $22 = (a+71d)+(a+111d) = 2a+182d = 2(a+91d)$. Thus, $a+91d = 11$ is the value of the middle term = the value of the average term, and the sum of all 183 terms is $183 \times 11 = \boxed{2013}$.

Method II: $a+(a+d)+(a+2d)+ \ldots +(a+182d) = 183a + \frac{183}{2}(182d) = 183(a+91d)$. As in Method I, $a+91d = 11$, so the sum is $183 \times 11 = 2013$.

Problem 3-6

Each of the 5 integers is paired with each of the other 4 integers, so each integer appears in 4 of the 10 sums. If we add all 10 sums together, their sum is 84. That means that the sum of the five integers is $84/4 = 21$. The smallest sum is 1, so the sum of the two smallest integers is 1. The largest sum is 15, so the sum of the two largest integers is 15. Clearly then, the middle integer is $21-15-1 = 5$. The second smallest sum has to be the sum of the smallest and middle integers, so the smallest integer is $4-5 = -1$. Similarly, the second largest sum, 14, is the sum of the largest and middle integers, so the largest integer is $14-5 = 9$. To get the second smallest integer, subtract the smallest integer from the sum of the two smallest integers; so the second smallest integer is $1-(-1) = 2$. Since the largest integer is 9 and the sum of the two largest integers is 15, the second largest integer is 6. The 5 integers are $\boxed{-1, 2, 5, 6, 9}$.