NEW YORK MATHEMATICS LEAGUE

P.O. Box 1090, Manhasset, New York 11030-8090

All official participants must take this contest at the same time.

Contest Number 2 *Any calculator without a QWERTY keyboard is allowed.* Answers must be exact *or* have 4 (or more) significant digits, correctly rounded. **November 12, 2013**

Name Answer Key **Teacher** ______ **Grade Level** ____ **Score** ____

Time Limit: 30 minutes NEXT CONTEST: DEC. 3, 2013 *Answer Column*

2-1. When written as 02-03-04, the date Feb. 3, 2004 consists of three consecutive integers whose sum is a perfect square. Writing your answer as MM-DD-YY, what is the first date after 02-03-04 that consists of three consecutive integers whose sum is a perfect square?

2-1. 11-12-13

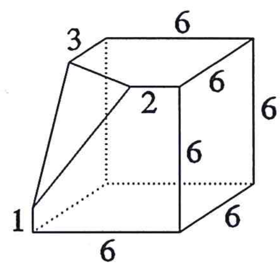
2-2. If $a \neq b$, but $a^2 + a = b^2 + b$, what is the value of $a + b$?

2-2. −1

2-3. What is the only odd prime factor of $2^{67} + 2^{71}$?

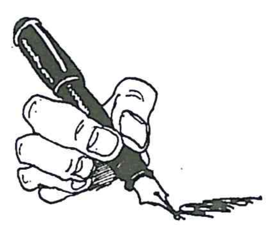
2-3. 17

2-4. The solid shown at the right was formed by making straight-line cuts through a corner of a cube of edge-length 6, then removing the corner. The distances of the three vertices of the soon-to-be-removed corner to the vertex of the cube nearest that corner were 1, 2, and 3, as shown. What is the volume of the solid remaining after the corner is removed?



2-4. 206

2-5. I wrote a list of 100 positive integers whose sum and product are equal. Of the integers on my list, at most how many can be a 1?



2-5. 98

2-6. In a certain quadrilateral, the three shortest sides are congruent, and both diagonals are as long as the longest side. What is the degree-measure of the largest angle of this quadrilateral?

2-6. 108

Problem 2-1

In any set of three consecutive integers, the middle integer is one-third the sum of the three integers. Since that sum must be divisible by 3, the required sum must be a perfect square greater than 9 (since $2+3+4 = 9$) that is divisible by 9. The least such sum is 36. The consecutive integers are 11, 12, and 13, and the date is $\boxed{11-12-13}$.

Problem 2-2

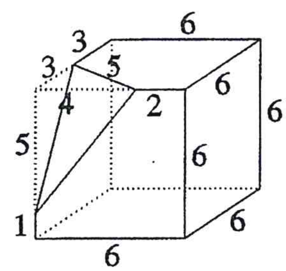
Since $a^2-b^2 = b-a$, we have $(a+b)(a-b) = -1(a-b)$. Since $a \neq b$, $(a-b) \neq 0$. Dividing the second equation through by $(a-b)$, we get $a+b = \boxed{-1}$.

Problem 2-3

Since $2^{67}+2^{71} = 2^{67}(1+2^4) = 2^{67}(17)$, the only odd prime factor of $2^{67}+2^{71}$ is $\boxed{17}$.

Problem 2-4

The volume of the cube is $6^3 = 216$. From the cube, a right pyramid was removed. The pyramid's base is a 3-4-5 right triangle, and the height of the pyramid is 5. The pyramid's base has area $bh/2 = (3\times 4)/2 = 6$. The volume is $Bh/3$, where B is the area of the base, so the volume of the pyramid is $(6\times 5)/3 = 10$. The volume of the remaining solid is $6^3-10 = 216-10 = \boxed{206}$.



Problem 2-5

Can the number of 1's be 100? No, the product is 1, but the sum is 100. Can the number of 1's be 99? No, the product is the value of the other number, and the sum is 99 more than that. Can the number of 1's be 98? Yes, since the sum of 98 1's, a 2, and a 100 is $98+2+100 = 200 = 1\times 1\times \ldots\times 1\times 2\times 100$ = the product. The maximum number of 1's is $\boxed{98}$.

[**NOTE:** To find all solutions with 98 1's, we can solve $98+x+y = xy$, or $98+x = y(x-1)$, so $y = (98+x)/(x-1) = (x-1+99)/(x-1) = 1 + 99/(x-1)$. Now, set $x-1$ equal, in turn, to each divisor of 99 and you will find all solutions.]

Problem 2-6

Since the diagonals of the quadrilateral are congruent, the upper two overlapping isosceles triangles are congruent to each other by SSS, so the four base angles, each marked with an x, are congruent. The lower two overlapping isosceles triangles are congruent to each other by SSS, so their vertex angles, each marked with a v, are congruent. Look at the upper triangle with an angle marked with an a. Its other two angles are congruent, since each is marked with an x. In the lower such triangle with an angle marked with an a, the other two angles are also congruent, so those two angles must both also be marked with an x, just like the upper such triangle. Finally, the base angles of the lower overlapping isosceles triangles are congruent. Since $v = x$, each base angle $b = x+v = 2x$. From the diagram, $10x = 360°$. Solving, $x = 36°$ and $3x = 108°$ or $\boxed{108}$.