

NEW YORK MATHEMATICS LEAGUE

P.O. Box 1090, Manhasset, New York 11030-8090

All official participants must take this contest at the same time.

Contest Number 1

Any calculator without a QWERTY keyboard is allowed. Answers must be exact or have 4 (or more) significant digits, correctly rounded.

October 15, 2013

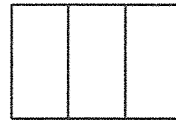
Name Answer Key Teacher _____ Grade Level _____ Score _____

Time Limit: 30 minutes

NEXT CONTEST: NOV. 12, 2013

Answer Column

1-1. Two congruent squares overlap to form 3 congruent, non-overlapping rectangles, as shown. If the perimeter of each of these rectangles is 18, what is the area of each?



1-1.

18

1-2. What is the greatest possible sum of two multiples of 12, each less than 100, whose greatest common factor is 24?

1-2.

168

1-3. The right side of the equation $3(ABC) = BBB$ represents a 3-digit number with 3 identical digits. If different letters represent different digits, what is the ordered triple of non-zero digits (A,B,C) ?

1-3.

(1, 4, 8)

1-4. If $N = 10^{2013} - 2013$ is expanded and written as an integer in standard form, what is the sum of the digits of N ?

1-4.

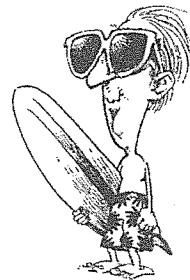
18112

1-5. What is the length of the hypotenuse of a right triangle in which the medians to the legs have lengths 3 and 4?

1-5.

$\sqrt{20}$

1-6. The Surfboard Store sells Special Surfboards that are made so carefully that only 1 in 1 thousand is bad. The store tests all Special Surfboards using a test that is 99% accurate. If my Special Surfboard tests bad, then, to the nearest 1%, what is the probability that my Special Surfboard is bad?



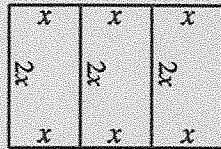
1-6.

9%

Eighteen books of past contests, *Grades 4, 5, & 6 (Vols. 1, 2, 3, 4, 5, 6)*, *Grades 7 & 8 (Vols. 1, 2, 3, 4, 5, 6)*, and *HS (Vols. 1, 2, 3, 4, 5, 6)*, are available, for \$12.95 each volume (\$15.95 Canadian), from Math League Press, P.O. Box 17, Tenafly, NJ 07670-0017.

Problem 1-1

The perimeter of each rectangle is 18, so $6x = 18$, and $x = 3$. The dimensions of each rectangle are 3 and 6, so the area of each is $3 \times 6 = \boxed{18}$.



Problem 1-2

The greatest common factor of the two chosen numbers is 24, so each must be a multiple of 24. The largest multiples of 24 that are less than 100 are $3 \times 24 = 72$ and $4 \times 24 = 96$. Their greatest common divisor is 24, and their sum is $3 \times 24 + 4 \times 24 = 7 \times 24 = \boxed{168}$.

Problem 1-3

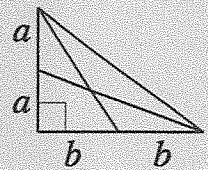
Since $BBB/3 = ABC$ is a 3-digit number that retains B as its middle digit and changes the other 2 digits, we can rule out $BBB = 333, 666, \text{ or } 999$. Now try each of $BBB = 111, 222, 444, 555, 777, \text{ and } 888$. The only solution of $ABC = BBB/3$ comes from $ABC = 444/3 = 148$, so $(A,B,C) = \boxed{(1,4,8)}$.

Problem 1-4

The number 10^{2013} has 2014 digits, 2013 of which are 0s. The result of the subtraction $10^{2013} - 2013$ is $1000 \dots 00 - 2013 = 999 \dots 997987$, a number with 2013 total digits. The sum of the last 4 digits is $7+9+8+7 = 31$. The first $2013 - 4 = 2009$ digits have a sum of $2009 \times 9 = 18081$. The sum of all 2013 digits is $31 + 18081 = \boxed{18112}$.

Problem 1-5

In the diagram, $a^2 + 4b^2 = 4^2$ and $4a^2 + b^2 = 3^2$. Adding, we get $5a^2 + 5b^2 = 16 + 9 = 25$. Dividing both sides by 5 and then multiplying both sides by 4, we get $4a^2 + 4b^2 = 20 = \text{hypotenuse}^2$, so hypotenuse's length is $\boxed{\sqrt{20}}$.



Problem 1-6

The 1 bad surfboard in 1000 will almost always test bad. Of the 999 good surfboards in 1000, 1% (about 10) typically test bad, even though they are good surfboards. Of the 11 surfboards that test bad, the expected number of bad surfboards is 1, and $1/11$ is about $\boxed{9\%}$.