# Chapter 12

# Sequences and Series

# Sequences & Series Notes & Homework Packet

## Lesson 1:

## Sequences

***Sequences*** are ordered lists of numbers. A sequence is formally defined as *a function that has its domain the set of positive integers,* i.e. {1, 2, 3, …, n}.

***Exercise 1:*** A sequence is defined by the equation $a_n = 2n - 1$.

(a) Find the first three terms of this sequence, denoted by $a_1$, $a_2$, and $a_3$.

(b) Which term has a value of 53?

(c) Explain why there will not be a term that has a value of 70.

***Exercise 2:*** A sequence is defined by the equation $a_n = \frac{n}{n+2}$. Find the first five terms of the sequence.

***Exercise 3:*** A sequence is defined by the equation $t_n = n^3$. Find the first six terms of this sequence.

***Exercise 4:*** Find the algebraic form for any term, $a_n$, to represent the following sequences.

(a) 5, 7, 9, 11, …

(b) 2, 4, 8, 16, …

(c) 10, 15, 20, 25, …

(d) 0, 3, 8, 15, …

(e) $1, \frac{1}{4}, \frac{1}{9}, \frac{1}{16}, \frac{1}{25}, \ldots$

(f) 1, -1, 1, -1, …

Recall that sequences can also be described using **______________________ *definitions***. When a *sequence is defined recursively, the terms are* _______________________________________________.

***Exercise 5:*** A sequence is defined by the recursive formula:

$a_n = a_{n-1} + 5$ with $a_1 = -2$.

Generate the first five terms of this sequence.

***Exercise 6:*** A sequence is defined by the recursive formula:

$a_n = (a_{n-1})^3 - 5$

$a_1 = -1$

Find the first four terms of this sequence.

***Exercise 7:*** Find the third term in the recursive sequence:

$$a_{k+1} = 2a_k + 8$$

$$a_1 = 7$$

***Exercise 8:*** For the recursively defined sequence, $t_n = (t_{n-1})^2 + 2$ and $t_1 = 2$, the value of $t_4$ is

(1) 18 (2) 38 (3) 456 (4) 1446

***Exercise 9:*** Determine a recursive definition, in terms of $a_n$, for the sequence shown below. Be sure to include a starting value.

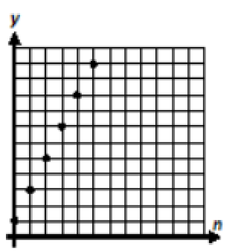
5, 10, 20, 40, 80, 160, …

***Exercise 10:*** Determine a recursive definition, in terms of $a_n$, for the sequence shown below. Be sure to include a starting value.

5, 12, 19, 26, …

***Exercise 11:*** Which of the following would represent the graph of the sequence $a_n = 2n + 1$? Explain your choice.

(1)

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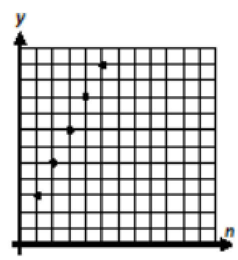


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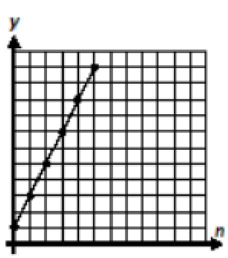


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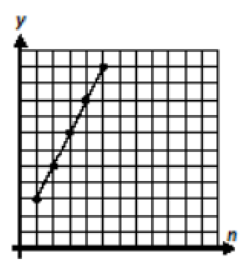


## Lesson 1:

## Sequences Homework

Answer all of the questions below.

1.) Given each of the following sequences defined by formulas, determine and label the first four terms.

(a) $a_n = 7n + 2$

(b) $t_n = \left(\frac{2}{3}\right)^n$

(c) $a_n = n^2 - 5$

(d) $t_n = \frac{1}{n+1}$

2.) Sequences below are defined recursively. Determine and label the ***next*** three terms of the sequence.

(a) $a_1 = 4; a_n = a_{n-1} + 8$

(b) $a_n = \frac{1}{2}a_{n-1}$ and $a_1 = 24$

(c) $b_n = b_{n-1} + 2n$ with $b_1 = 5$

(d) $t_n = 2t_{n-1} - n^2$ and $t_1 = 4$

3.) What is the seventh term of this sequence? $a_1 = 3$, $a_n = a_{n-1} + 6$

4.) Given the sequence 7, 11, 15, 19, …, which of the following represents a formula that will generate it?

(1) $a_n = 4n + 7$ (2) $a_n = 3n + 4$ (3) $a_n = 3n + 7$ (4) $a_n = 4n + 3$

5.) Which of the following formulas would represent the sequence 10, 20, 40, 80, 160, …

(1) $a_n = 10^n$ (2) $a_n = 10(2)^n$ (3) $a_n = 5(2)^n$ (4) $a_n = 2n + 10$

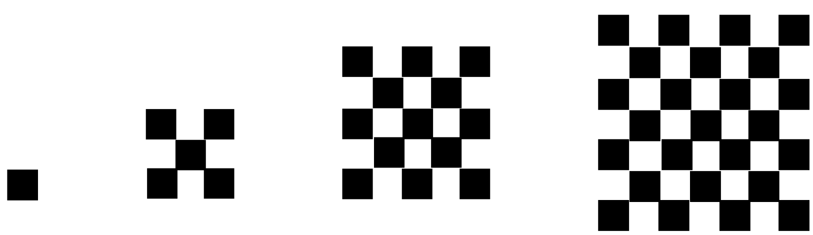
6.) For each of the following sequences, determine the algebraic formula, that defines the sequence. ***Do not write it using the recursive formula***.

(a) 5, 10, 15, 20, … (b) 3, 9, 27, 81, … (c) $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \ldots$

7.) For each of the follow sequences, state a recursive definition. Be sure to include a starting value.

(a) 8, 6, 4, 2, … (b) 2, 6, 18, 54, … (c) 2, -2, 2, -2, …

8.) A tiling pattern is created from a single square then expanded as shown. If the number of squares in each pattern defines a sequence, then determine the number of squares in the seventh pattern. Explain how you arrived at your choice. Write a recursive definition for the pattern.



<u>**Lesson 2:**</u>

<u>**Arithmetic Sequences**</u>

In Common Core Algebra 1, you studied two particular sequences known as arithmetic (based on constant ________________________ to get the next term) and geometric (based on constant ________________________ to get the next term).

**Arithmetic Sequence Recursive Definition**

Given $a_1$ then ________________________________________________

where *d* is called the ***common*** _______________ and can be positive or negative.

***Exercise 1:*** Generate the next three terms of the given arithmetic sequences.

(a) $a_n = a_{n-1} + 6$ with $a_1 = 2$

(b) $a_n = a_{n-1} + \frac{1}{2}$ and $a_1 = \frac{3}{2}$

***Formula for Arithmetic Sequence:***

Any term, $a_n$, of an arithmetic sequence, can be found by using a formula. This formula is given on the Regents exam.

***Formula:*** $a_n$ = ____________________________________________

$a_n$ = _______________

$a_1$ = _______________ term in the sequence

n = _______________ of terms

d = common _______________

***Exercise 2:*** For some number $t$, the first three terms of an arithmetic sequence are $2t$, $5t - 1$, and $6t+2$.

(a) What is the value of $t$?

(b) What is the numerical value of the fourth term?

***Exercise 3:*** Given that $a_1 = 6$ and $a_4 = 18$ are members of an arithmetic sequence, determine the value of $a_{20}$.

***Exercise 4:*** In an arithmetic sequence $t_n = t_{n-1} - 5$. If $t_1 = 3$ determine the values of $t_6$ and $t_{25}$. Show calculations that lead to your answers.

***Exercise 5:*** Find the number of terms in an arithmetic sequence whose first two terms are -3 and 4, and whose last term is 116.

***Exercise 6:*** In an arithmetic sequence, $a_4 = 3$ and $a_7 = 18$. Determine the formula for $a_n$, the $n^{th}$ term of this sequence.

***Exercise 7:*** In an arithmetic sequence, $a_5$ = -2 and $a_9$ = -10. Determine the formula for $a_n$, the $n^{th}$ term of this sequence.

***Exercise #8:*** Find the $62^{nd}$ term of an arithmetic sequence whose first term of -3, with a common difference of 4.

# Lesson 2:

## Arithmetic Sequences Homework

Answer all questions below.

1.) Generate the next *three* terms of each arithmetic sequence shown below.

(a) $a_1 = -2$ and d = 4

(b) $a_1 = 3$, $a_2 = 1$

2.) In an arithmetic sequence $t_n = t_{n-1} + 7$. If $t_1 = -5$ determine the values of $t_4$ and $t_{20}$. Show the calculations that lead to your answers.

3.) If $x + 4$, $2x + 5$, and $4x + 3$ represent the first three terms of an arithmetic sequence, then find the value of *x*. What is the fourth term?

4.) If $a_1 = 12$ and $a_n = a_{n-1} - 4$, then which of the following represents the value of $a_{40}$?

(1) -148

(2) -140

(3) -144

(4) -172

5.) In an arithmetic sequence of numbers $a_1$ = -4 and $a_6$ = 46. Which of the following is the value of $a_{12}$?

(1) 120 (2) 146 (3) 92 (4) 106

6.) The first term of an arithmetic sequence whose common difference is 7 and whose 22nd term is given by $a_{22}$ = 143 is which of the following?

(1) -25 (2) -4 (3) 7 (4) 28

7.) What is the common difference of the arithmetic sequence below?

$-6x, -x, 4x, 9x, 14x, \ldots$

(1) $-5$ (2) $-5x$ (3) 5 (4) $5x$

8.) What is the formula for the *n*th terms of sequence *B* shown below?

*B* = 10, 12, 14, 16, …

(1) $b_n = 8 + 2n$

(2) $b_n = 10 + 2n$

(3) $b_n = 10(2)^n$

(4) $b_n = 10(2)^{n-1}$

**Lesson 3:**

**Geometric Sequences**

***Geometric Sequences*** are defined very similarly to arithmetic, but with a ______________ constant instead of an ______________ one.

***Geometric Sequence Recursive Definition:***

Given $a_1$, then $a_n =$ ______________________

where *r* is the *common* ______________ and can be positive or negative and is often a _____________.

***Exercise 1:*** Generate the next three terms of the geometric sequences given below.

(a) $a_1 = 4$ and $r = 2$

(b) $a_n = a_{n-1} \cdot \frac{1}{3}$ with $a_1 = 9$

(c) $t_n = t_{n-1}\sqrt{2}$ with $t_1 = 3\sqrt{2}$

Like an arithmetic sequence, we also need to be able to determine any given term of a *geometric sequence* based on the first value, the *common* ______________, and the index. Again, we are given this formula on the Regents Exam.

***Formula:*** $a_n = a_1 r^{n-1}$

***Exercise 2:*** Find the fifth term of a geometric sequence whose first term is $\frac{1}{3}$ and whose common ratio is -6.

***Exercise 3:*** Given that $a_1 = -2$ and $a_2 = 8$ are the first two terms of a geometric sequences, determine the values $a_4$ and $a_8$. Show calculation that lead to your answer.

***Exercise 4:*** What is the common ratio of the geometric sequence shown below?

-2, 4, -8, 16, …

***Exercise 5:*** What is the common ratio of the geometric sequence whose first term is 27 and fourth term is 64?

***Exercise 6:*** What is the common ratio of the geometric sequence whose first term is -7 and whose fourth term is 189?

***Exercise 7:*** What is the formula for the n^th^ term of the sequence 54, 18, 6, …?

(1) $a_n = 6\left(\frac{1}{3}\right)^n$  (2) $a_n = 6\left(\frac{1}{3}\right)^{n-1}$  (3) $a_n = 54\left(\frac{1}{3}\right)^n$  (4) $a_n = 54\left(\frac{1}{3}\right)^{n-1}$

***Exercise 8:*** Write a formula to find $a_n$, the n^th^ term of the infinite sequence:

$$\frac{1}{5}, -\frac{1}{25}, \frac{1}{125}, -\frac{1}{625}, \dots$$

***Exercise 9:*** What is the common ratio of the geometric sequence whose first term is 5500 and whose fourth term is 44.

***Exercise 10:*** What is the common ratio of the sequence $-\frac{1}{2}, \frac{3}{4}, -\frac{9}{8}$?

## Lesson 3:

## Geometric Sequences Homework

Answer all questions below. Show work when necessary.

1.) In a geometric sequence, it is known that $a_1 = -1$ and $a_4 = 64$. The value of $a_{10}$ is

(1) -65,536 (2) 262,144 (3) 512 (4) -4096

2.) Generate the next **three** terms of each geometric sequence defined below.

(a) $a_1 = -8$ with $r = -1$

(b) $a_n = a_{n-1} \cdot \frac{3}{2}$ and $a_1 = 16$

3.) Given that $a_1 = 5$ and $a_2 = 15$ are the first two terms of a geometric sequence, determine the values of $a_3$ and $a_{10}$. Show the calculations that lead to your answers.

4.) What is the fifteenth term of the sequence 5, -10, 20, -40, 80, … ?

5.) What is the common ratio in the geometric sequence whose first term is $\frac{1}{5}$ and whose fifth term is 125?

6.) What term of the geometric sequence 2187, 729, 243, …. is 27?

7.) Find the common ratio of the geometric sequence whose first term is 4 and whose 3rd term is 36.

**Lesson 4:**

**Summation Notation**

Sometimes we want to find the _______________ of the terms of a sequence. We can do this by using *Sigma Notation*. (Sigma means _______________).

**To evaluate:** Substitute all consecutive integers from the bottom number of the Sigma, ending at the top number of the Sigma. Then add up all of the terms.

"The summation from 1 to 4 of 3n"

# to end at -->

Sigma Symbol -->

# to start at -->

$$\sum_{n=1}^{4}(3n)$$

***Exercise 1:*** Evaluate each of the following sums.

(a) $\sum_{i=3}^{5} 2i$

(b) $3 + \sum_{k=-1}^{3} k^2$

(c) $\sum_{i=1}^{3} i(i+1)$

(d) $\sum_{i=1}^{5} (-1)^i$

***Exercise 2:*** Which of the following represents the value of $\sum_{i=1}^{4}\frac{1}{i}$?

(1) $\frac{1}{10}$ (2) $\frac{9}{4}$ (3) $\frac{25}{12}$ (4) $\frac{31}{24}$

***Exercise 3:*** Simplify: $\sum_{a=1}^{4}(x-a^2)$

***Exercise 4:*** Find, in terms of x, the value of $\sum_{k=2}^{6}(2k+x)$

***Exercise 5:*** Consider the sequence defined recursively by $a_n = a_{n-1} + 2a_{n-2}$ and $a_1 = 0$ and $a_2 = 1.$ Find the value of $\sum_{i=4}^{7} a_i$.

***Exercise 6:*** Express each sum using sigma notation. Use *n* as your index variable. First, consider any patterns you notice amongst the terms involved in the sum. Then, work to put these patterns into a formula and sum.

(a) 9 + 16 + 25 + … + 100

(b) -6 + -3 + 0 + 3 + … + 15

(c) $\frac{1}{25} + \frac{1}{5} + 1 + 5 + \cdots + 625$

***Exercise 7:*** Which of the following represents the sum 3 + 6 + 12 + 24 + 48?

(1) $\sum_{i=1}^{5} 3^i$

(2) $\sum_{i=0}^{4} 3(2)^i$

(3) $\sum_{i=0}^{4} 6^{i-1}$

(4) $\sum_{i=3}^{48} i$

**Lesson 4:**

**Summation Notation Homework**

Complete the questions below.

1.) Express the sum $\frac{2}{3}+\frac{3}{4}+\frac{4}{5}+\frac{5}{6}+\frac{6}{7}$ using sigma notation.

2.) Ms. Hill asked her students to express the sum 1 + 3 + 5 + 7 + 9 + . . . + 39 using sigma notation. Four different student answers were given. Which student answer is correct?

(1) $\sum_{k=1}^{20}(2k-1)$

(2) $\sum_{k=2}^{40}(k-1)$

(3) $\sum_{k=-1}^{37}(k+2)$

(4) $\sum_{k=1}^{39}(2k-1)$

3.) Which expression is equivalent to the sum of the sequence 6, 12, 20, 30?

(1) $\sum_{n=4}^{7} 2^n - 10$

(2) $\sum_{n=2}^{5} 5n - 4$

(3) $\sum_{n=3}^{6} \frac{2n^2}{3}$

(4) $\sum_{n=2}^{5} n^2 + n$

4.) Which of the following represents the sum 2 + 5 + 10 + … + 82 + 101?

(1) $\sum_{j=1}^{6}(4j-3)$

(2) $\sum_{j=3}^{103}(j-2)$

(3) $\sum_{j=1}^{10}(j^2+1)$

(4) $\sum_{j=0}^{11}(4^j+1)$

5.) Determine the value of $\sum_{x=0}^{2}(1-2a)^x$, leave your answer in terms of a.

6.) What is the expression $2+\sum_{k=1}^{3}2(k-x)$ equal to? Leave your answer in simplest form.

7.) Apply Sigma Notation. Leave your answers in simplest form.

(a) $\sum_{k=0}^{3}(3k-2)^2$

(b) $\frac{1}{2}\sum_{x=2}^{5}x^2$

8.) Write each of the following sums using sigma notation. Use *k* as your index variable. There may be more than one correct way to write each sum.

(a) $2 + 4 + 8 + \ldots + 64 + 128$

(b) $\frac{1}{1}+\frac{1}{4}+\frac{1}{9}+\cdots+\frac{1}{81}+\frac{1}{100}$

## Lesson 5:

## Arithmetic Series

A ______________ is simply the ____________ ***of the terms of a sequence***. The fundamental definition/notation of a series is below.

### THE DEFINITION OF A SERIES

If the set $\{a_1, a_2, a_3, ...\}$ represent the elements of a sequence then the series, $S_n$, is defined by:

$$S_n = \sum_{i=1}^{n} a_i$$

***Exercise 1:*** Given the arithmetic sequence defined by $a_1 = -2$ and $a_n = a_{n-1} + 5$, then which of the following is the value of $S_5 = \sum_{i=1}^{5} a_i$?

(1) 32 (2) 40 (3) 25 (4) 27

The *sums associated with arithmetic sequences, known as* ______________ ***series***, have a formula that we can use. This formula is ***not*** given to you on the Regents Exam, so it must be memorized!

***Arithmetic Series Formula:***

***Exercise 2:*** Find the sum of the first 24 terms of the sequence 3, 7, 11, 15, …

***Exercise 3:*** Which of the following is the sum of the first 100 natural numbers?

(1) 5,000 (2) 5,100 (3) 10,000 (4) 5,050

***Exercise 4:*** Find the sum of each arithmetic series described or shown below.

**Recall:** ________________________________

(a) The sum of the sixteen terms given by:

-10 + -6 + -2 + … + 46 + 50

(b) The first term is -8, the common difference, d, is 6 and there are 20 terms.

(c) The last term is $a_{12} = -29$ and the common difference, d, is -3.

(d) The sum 5 + 8 + 11 + … + 77.

***Exercise 5:*** The first and last terms of an arithmetic series are 7 and -121, respectively, and the series has a sum of -1026. How many terms are in this series?

***Exercise 6:*** Kirk has set up a college savings account for his son, Maxwell. If Kirk deposits $100 per month in an account, increasing the amount he deposits by $10 per month, then how much will be in the account after 10 years?

***Exercise 7:*** An auditorium has 42 rows of sets. The first row has 5 seats, and each succeeding row has three more seats than the previous row. How many seats are in the auditorium?

## Lesson 5:

## Arithmetic Series Homework

Answer all of the questions below. Show all work.

1.) What is the sum of the first 19 terms of the sequence 3, 10, 17, 24, 31, … ?

(1) 1188 (2) 1197 (3) 1254 (4) 1292

2.) Find the sum of the first 15 terms of the sequence -8, -2, 4, 10, …

(1) 1020 (2) 516 (3) 504 (4) 510

3.) The sum of the first 50 natural numbers is

(1) 1275 (2) 1875 (3) 1250 (4) 950

4.) The first and last terms of an arithmetic series are 5 and 27, respectively, and the series has a sum of 192, then the number of terms in the series is

(1) 18 (2) 11 (3) 14 (4) 12

5.) Determine the sum of the first twenty terms of the sequence whose first five terms are 5, 14, 23, 32, 41.

6.) Arlington High School recently installed a new black-box theatre for local productions. They only had room for 14 rows of seats, where the number of seats in each row constitutes an arithmetic sequence starting with eight seats and increasing by two seats per row thereafter. How many seats are in the new black-box theatre? Show the calculations that lead to your answer.

7.) Simon starts a retirement account where he will place $50 into the account on the first month and increase his deposit by $5 per month each month after. If he saves this way for the next 20 years, how much will the account contain in principal?

# Lesson 6:

## Geometric Series

Just as we can sum the terms of an arithmetic sequence to generate an arithmetic series, we can also sum the terms of a geometric sequence to generate a ***geometric series***.

***Exercise 1:*** Given a geometric series defined by the recursive formula $a_1 = 3$ and $a_n = a_{n-1} \cdot 2$, which of the following is the value of $S_5 = \sum_{i=1}^{5} a_i$?

(1) 106 (2) 75 (3) 93 (4) 35

The formula for a ***geometric series*** is given to us on the Regents Exam:

$$S_n = \frac{a_1 - a_1 r^n}{1 - r}$$

Where: $a_1$ = first term

r = common ratio

n = $n^{th}$ term

***Exercise 2:*** Which of the following represents the sum of a geometric series with 8 terms whose first term is 3 and whose common ratio is 4?

(1) 32,756 (2) 28,765 (3) 42,560 (4) 65,535

***Exercise 3:*** Find the value of the geometric series shown below. Show calculations that lead to your final answer.

$$6 + 12 + 24 + \ldots + 768$$

***Exercise 4:*** Find the sum of the first 8 terms of the sequence:

$$\frac{1}{5}, \frac{2}{5}, \frac{4}{5}, \frac{8}{5}, \ldots$$

***Exercise 5:*** A geometric series has a first term of 8 and a last term of $\frac{1}{16}$. Its common ratio is $\frac{1}{2}$. Find the value of this series.

***Exercise 6:*** A person places 1 penny in a piggy bank on the first day of the month, 2 pennies on the second day, 4 pennies on the third, and so on. Will this person be a millionaire at the end of a 31 day month? Show the calculations that lead to your answer.

***Exercise 7:*** Charlotte earns $54,000 during her first year as an accountant and earns 3.5% increase in each successive year.

(a) Write a geometric series formula, $S_n$, for Charlotte's earnings over *n* years.

(b) Use this formula to find Charlotte's total earnings for her first 8 years working as an accountant, to the nearest cent.

***Exercise 8:*** Hope earns \$1000 a year. She gets a 6% raise every year.

(a) Write a geometric series formula, $S_n$, for the amount she earns over *n* years.

(b) Write this formula to find the amount of money, to the nearest dollar, that Hope earns after 18 years.

# Lesson 6:

## Geometric Series Homework

Answer all questions below.

1.) The sum of the first eight terms of the series 3 – 12 + 48 – 192 + … is

(1) -13,107 (2) -21,845 (3) -39,321 (4) -65,535

2.) Find the sum of the first 8 terms of the sequence 1, 6, 36, 216, …

3.) Find the sums of geometric series with the following properties:

(a) $a_1 = 6, r = 3 \text{ and } n = 8$

(b) $a_1 = 20, r = \frac{1}{2}, \text{ and } n = 6$

4.) If the geometric series $54 + 36 + \cdots + \frac{128}{27}$ has seven terms in its sum then the value of the sum is

(1) $\frac{4118}{27}$ (2) $\frac{1274}{3}$ (3) $\frac{1370}{9}$ (4) $\frac{8241}{54}$

5.) A geometric series has a first term of 32 and a final term of $-\frac{1}{4}$ and a common ratio of $-\frac{1}{2}$. The value of this series is

(1) 19.75 (2) 16.25 (3) 22.5 (4) 21.25

6.) Find the sum of the geometric series shown below. Show the work that leads to your answer.

$$27+9+3+\cdots\frac{1}{729}$$

7.) Alex earns $40,000 during her first year as a teacher and earns 3.2% increase in each successive year.

(a) Write a geometric series formula, $S_n$, for Alex's earnings over *n* years.

(b) Use this formula to find Alex total earnings for her first 8 years working as an accountant, to the nearest cent.

## Lesson 7

## Mortgage Payments

***Mortgages*** are not just made on houses. They are large amounts of money borrowed from a bank on which interest is calculated. The interest is calculated on a regular basis (usually monthly). Regular payments are made on the amount of money owed so that over time the principal (original amount borrowed) is paid off as well as any interest on the amount owed.

This is a complex process that involved geometric series.

***Formula:***

$$m = \frac{P\left(\frac{r}{12}\right)}{1-\left(1+\frac{r}{12}\right)^{-n}}$$

*m* = mortgage payment (monthly)

*p* = principal (loan amount)

*r* = interest rate (decimal)

*n* = number of payments

***Examples:***

1.) Tom's house will cost $300,000. He needs to pay a deposit of 10% and will pay the remaining 90% over 30 years at 8% interest rate per year. How much does Tom need per month to buy this house?

2.) Calculate the monthly payment needed to pay off a $200,000 loan at 4% yearly interest rate over a 20 year period. Now calculate the pay off period to be 30 years. How much less is the monthly payment?

3.) You would like to buy a home priced at $200,000. You plan to make a payment of 10% of the purchase price.

(a) Compute the total monthly payment for a 30-year mortgage at 4.8% annual interest.

(b) What is the total interest paid over the life of the loan?

(c) Compute the total monthly payment and the total interest paid over the life of a 20-year mortgage at 4.8% annual interest.

(d) Why would someone choose a 20-year mortgage over a 30-year mortgage? Why might another person choose a 30-year mortgage?

4.) Suppose you would like to buy a home priced at $180,000. You qualify for a 30-year mortgage at 4.5% annual interest.

(a) Calculate the total monthly payment and the interest paid over the life of the loan if you make a 3% down payment.

(b) Calculate the total monthly payment and the interest paid over the life of the loan if you make a 10% down payment.

(C) Calculate the total monthly payment and the interest paid over the life of the loan if you make a 20% down payment.

5.) The following amortization table shows the amount of payments to principal and interest on a $100,000 mortgage at the beginning and end of a 30-year loan.

| Month/ Year | Payment | Principal Paid | Interest Paid | Total Interest | Balance |
|---|---|---|---|---|---|
| Sept. 2014 | $ 477.42 | $ 144.08 | $ 333.33 | $ 333.33 | $ 99,855.92 |
| Oct. 2014 | $ 477.42 | $ 144.56 | $ 332.85 | $ 666.19 | $ 99,711.36 |
| Nov. 2014 | $ 477.42 | $ 145.04 | $ 332.37 | $ 998.56 | $ 99,566.31 |
| Dec. 2014 | $ 477.42 | $ 145.53 | $ 331.89 | $ 1,330.45 | $ 99,420.78 |
| Jan. 2015 | $ 477.42 | $ 146.01 | $ 331.40 | $ 1,661.85 | $ 99,274.77 |
| Mar. 2044 | $ 477.42 | $ 467.98 | $ 9.44 | $ 71,845.82 | $ 2,363.39 |
| April 2044 | $ 477.42 | $ 469.54 | $ 7.88 | $ 71,853.70 | $ 1,893.85 |
| May 2044 | $ 477.42 | $ 471.10 | $ 6.31 | $ 71,860.01 | $ 1,422.75 |
| June 2044 | $ 477.42 | $ 472.67 | $ 4.74 | $ 71,864.75 | $ 950.08 |
| July 2044 | $ 477.42 | $ 474.25 | $ 3.17 | $ 71,867.92 | $ 475.83 |
| Aug. 2044 | $ 477.42 | $ 475.83 | $ 1.59 | $ 71,869.51 | $ 0.00 |

(a) Describe the changes in the amount of principal paid each month as the end of the loan approaches.

(b) Describe the changes in the amount of interest paid each month as the end of the loan approaches.

**Lesson 7**

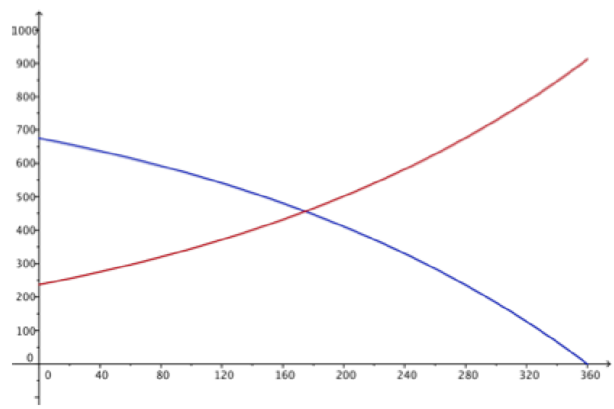
**Mortgage Payments Homework**

Complete the following questions. Show all work, including formulas.

1.) Christopher wants to buy a $200,000 home with a 30-year mortgage at 4.5% annual interest paying 10% down.

(a) What is the monthly payment on the house?

(b) The graph below depicts the amount of your payment from part (b) that goes to the interest on the loan and the amount that goes to the principal on the loan. Explain how you can tell which graph is which.



2.) In the summer of 2014, the average listing price for homes for sale in the Hollywood Hills was $\$2,663,995$.

Suppose you want to buy a home at that price with a $30$-year mortgage at $5.25\%$ annual interest, paying $10\%$ as a down payment. What is your total monthly payment on this house?

3.) Suppose that you want to buy a house that costs $175,000. You can make a 5% down payment, and

a. Find the monthly payment for a 30-year mortgage on this house at a 4.25% interest rate.

b. Find the monthly payment for a 15-year mortgage on this house at the same interest rate.

***Challenge Problem from Common Core Sampler:***

4.) Monthly mortgage payments can be found using the formula below:

$$M = \frac{P\left(\frac{r}{12}\right)\left(1+\frac{r}{12}\right)^{n}}{\left(1+\frac{r}{12}\right)^{n}-1}$$

M = monthly payment

P = amount borrowed

r = annual interest rate

n = number of monthly payments

The Banks family would like the borrow $120,000 to purchase a home. They qualified for an annual interest rate 4.8%. Algebraically determine the fewest number of whole years the Banks family would need to include in the mortgage agreement in order to have a monthly payment of no more than $720.